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Vagaries of the Euro: an Introduction to ARIMA Modeling

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The goal of this study is to provide a detailed example of ARIMA modeling in the form of a case study that could be used for teaching and learning purposes. We present three ARIMA models using macroeconomic indicators to model the USD/EUR exchange rate. We find that over the period from January 1994 to October 2007, the monthly USD/EUR exchange rate is best modeled by a linear relationship between its past three values and the current and past three values of the difference between the log-levels of the share prices indices in the European Monetary Union and the USA.

Keywords: *ARIMA models, time series, exchange rates, macroeconomic model.*

Introduction

The goal of this paper is to provide a practical and accessible example of ARIMA modeling, which can then serve as a case study for students in econometrics or statistics. We chose to develop our case on the exchange rate between the US dollar and the Euro (USD/EUR) because this field of study has two main advantages. First, up-to-date data, as the ones we used in this paper, are available freely to download from international and national agencies. Second, exchange rates have been extensively studied in the literature, and several competing economic theories have been put forth to explain their fluctuations in terms of macro-economic variables. Thus, it gives an excellent opportunity, as well as an example for students, to study one possible method to put those theories to test.

We think that these combined advantages provide both teachers and students with a perfect framework to practice ARIMA modeling anew. New up-to-date data which could be tied to current events will always be freely available, while data from the period this paper is concerned with will remain available for replication purposes. Readers will not find in the following pages a new theory concerning the movements of exchange rates, though the results of our models still have interesting theoretical implications. Indeed, our results build on the strength of ARIMA modeling to provide insights on the structure of the USD/EUR time series. Our models also put to test the influence on exchange rates of macroeconomic covariates whose choices have been theoretically driven. As we will show, exchange rates modeling and forecasting well deserve their reputation as difficult endeavors.

The paper is developed in the following way. In the first part, we present the different time series we consider, explaining our choices for the covariates. In the second part, we explain our methodology and modeling approach, referring to the general methodology of ARIMA modeling. Then, we present the resulting models, and their different characteristics. Finally, the discussion section places our models in a broader perspective, and relates the equations to practical explanations.

Context and Background

In this section, we provide some historical as well as economic context to our exercise in ARIMA modeling. To help with understanding the special role of the USD/EUR exchange rate, we first give a brief overview of the historical developments that lead to the inception of the European currency, before providing the rationale behind our choice of covariates for the study in a second section.

Presenting the euro: A short history of the European currency

Exchange rates, and thus an understanding of their fluctuations, are at the crossroad of geopolitical and economic histories of our modern societies. It is not the subject of this paper to delve into their fascinating intricacies. However, because the data in this paper cover an important period in recent economic history which oversaw the convergence of the monetary systems in Western Europe and the inception of the Euro, it is important to have a few facts in mind. The idea of a monetary system common to the countries of Western Europe date back to the late 1970s, and the European Currency Unit (ECU) was created on March 13, 1979 as an internal account unit for the European Community. In June 1988, the European Council confirmed the objective of a progressive realization of an Economic and Monetary Union (EMU) in Western Europe. A year later, in June 1989, it further decided that this progressive realization would unravel in three stages: from July 1st, 1990 to December 31st, 1993; then, from January 1st, 1994 to December 31st, 1998; and finally, from January 1st, 1999 to nowadays (European Central Bank, 2008). Since the beginning of this three-stage plan, the use of the ECU, previously restricted, was freely granted, and it began to be more commonly used for some international financial transactions. The second stage marks a strengthening in the convergence of the different monetary systems and the progressive emancipation of the central banks from national sovereignties. It is also during this period that the European Central Bank (ECB), as well as the European Monetary Institute, were created. Finally, the last stage is synonymous with the introduction of the Euro (EUR) to replace the ECU as an accounting unit as of January 1st, 1999, and as a real currency with the introduction of coins and banknotes as of January 1st, 2002.

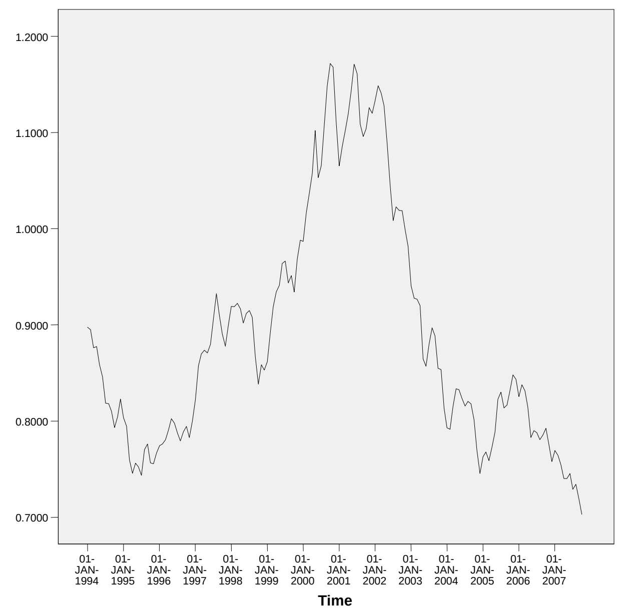

Figure 1. Exchange rate, EUR per USD, January 1994 to September 2007

These dates are important to our study for several reasons. The first thing to recognize is that our dataset is constituted of monthly time series that cover the period from January 1994 to October 2007. Hence, the exchange rate from January 1994 to December 1998 is calculated from quotations on the ECU, at the time the only European currency. If the ECU was a "virtual" currency constituted as a weighted average of the actual currencies of the members of the EMU, the convergence imposed by stage II of the European Council plan ensures some validity to the extrapolation of the exchange rate time series to the pre-euro period. Moreover, the calculations were undertaken by the ECB, the original source of the time series in this dataset. From a global point of view, the progressive introduction of a European currency, backed by an independent European central bank, has been one main driver to the situation nowadays.

It is almost unnecessary to cite the importance to the world's economy that the euro/USdollar exchange rate has acquired. Recently, using economic projections for

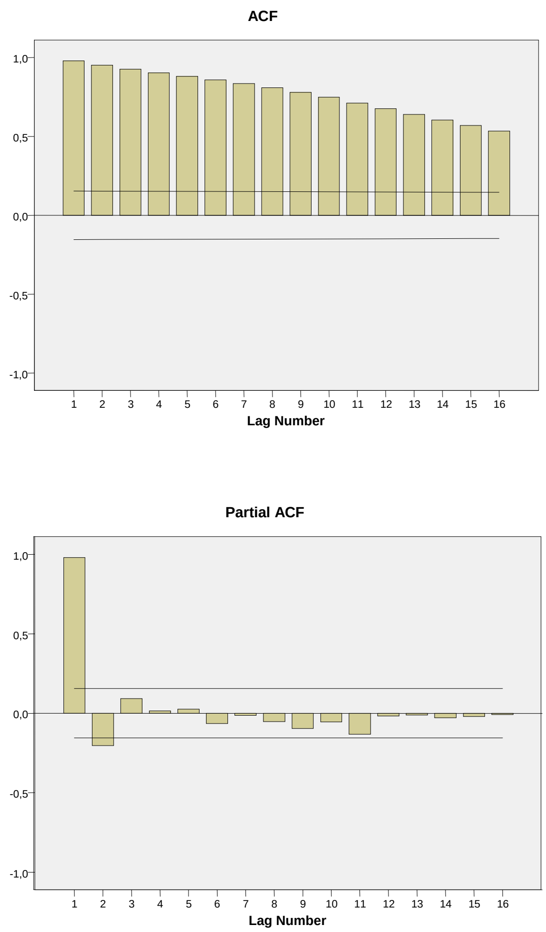

Figure 2. EUR per USD: sample Autocorrelation Function (top); sample Partial Autocorrelation Function (bottom)

the next 15 years, some authors (e.g. Chinn & Frankel, 2006) have argued that the chance that the Euro currency will take over the role of the world's reserve currency from the US Dollar is not null. Moreover, the recent turmoil on the financial markets, in February-March 2008, may be seen as a sign of a weakening of the US Dollar against the Euro – one of the scenarios used by Chinn and Frankel for their projections.

From a purely econometric point of view, the exchange rate displayed in Figure 1, presents some characteristics of a (non stationary) drifted random walk, with two main periods corresponding to a weakening of EUR against USD from 1995 to 2001, followed by the reverse phenomenon from 2001 to nowadays. Furthermore, the sample Autocorrelation Function (ACF) and the sample Partial Autocorrelation Function (PACF), which are representative of the correlation structure across periods of the time series, displayed in Figure 2 for the

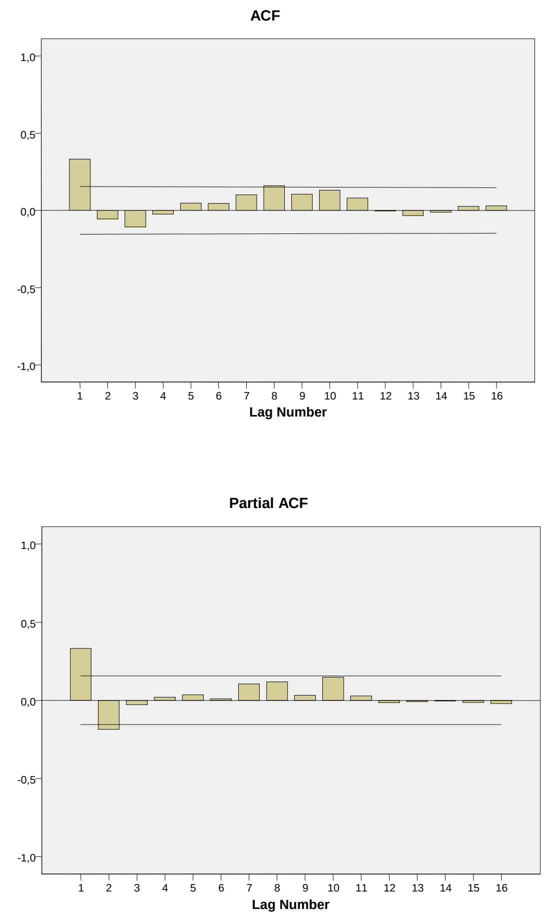

Figure 3. Change in EUR per USD: sample Autocorrelation Function (top); sample Partial Autocorrelation Function (bottom)

EUR-per-USD time series and in Figure 3 for the first difference of the time series, again show some characteristics of a non-stationary process. That is, the sample ACF of exchange rate decays very slowly, while the corresponding PACF shows only one very significant contribution at the first lag. At the same time, for the time series of the first differences, the ACF cuts off after the first lag, while the PACF cuts off after only two significant contributions.

The choice of covariates: choosing economic indicators

The goal of this section is to present each of the covariates that enter the study below in the economic framework that lead us to choose it. For this purpose, we will appeal to five different economic constructs: the Interest Rate Parity (IRP), Purchasing Power Parity (PPP), Money Aggregates, and Business Cycles.

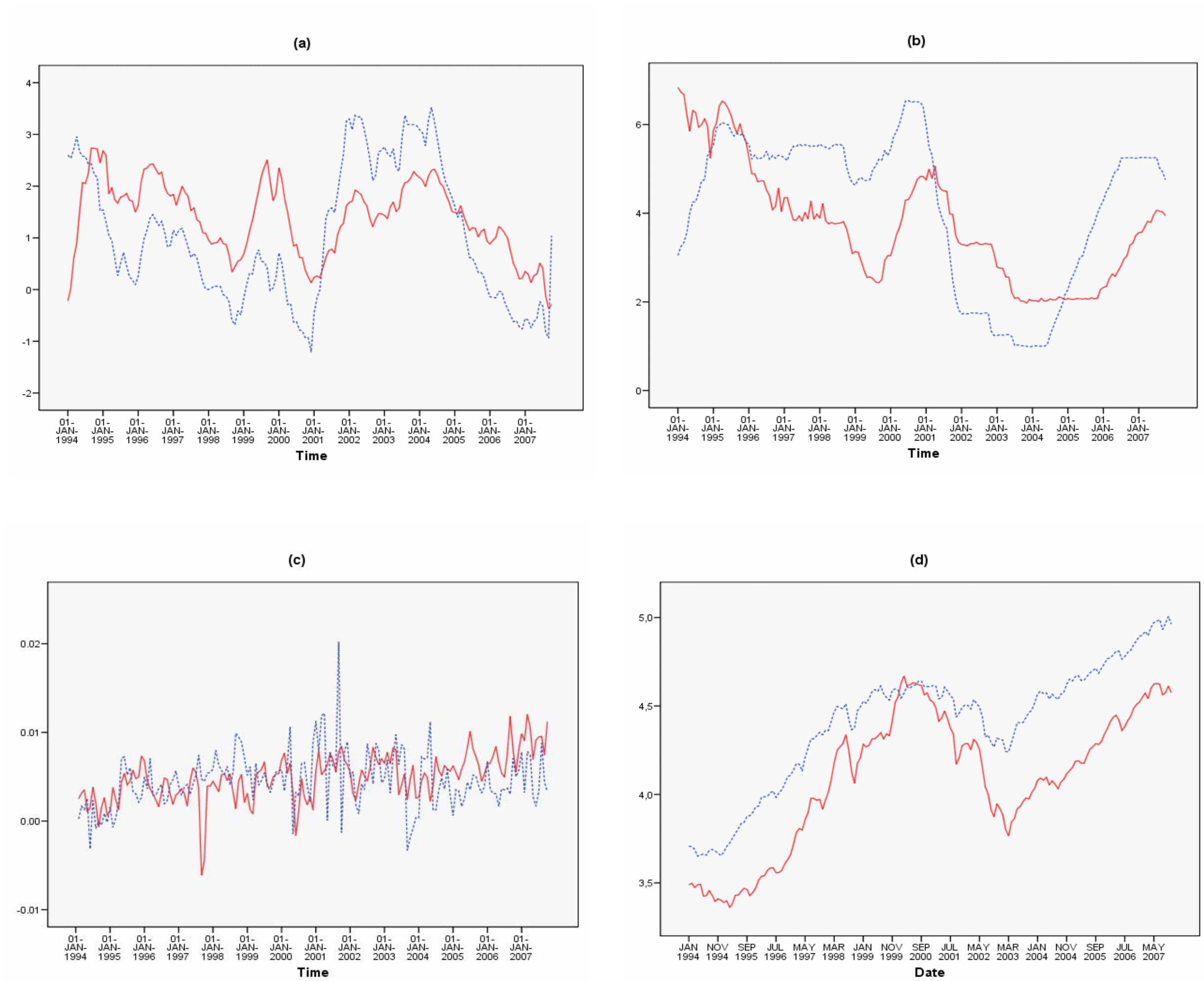

Figure 4. (a) and (b): Interest Rate Parity covariates: (a) Slopes of Interest rates; (b) Immediate Interest Rates --- (c) log-changes in M3 aggregates --- (d) Business Cycles: log-levels of share prices indices --- Legend: European Monetary Union: continuous/red line; USA: dashed/blue line

Interest Rate Parity (IRP)

Interest Rate Parity is a fundamental relationship that relates interest rates and exchange rates. It relies on the idea of a possible financial arbitrage which makes use of the differential in interest rates between two currencies. The idea is to borrow money in the currency with the lower interest rate and invest it in the currency with the higher interest rate. Since a consistent arbitrage would ultimately lead to infinite wealth, there must be a mechanism that balances the interest rates differential. In this theory, the exchange rate is seen as the balance mechanism. Conversely, since this arbitrage should not exist, interest rates can be used to forecast the movements of the exchange rate between the two currencies.

The classical approach to exchange rate forecasting using IRP belongs to the Chicago tradition (cf. Frankel, 1979). Under this approach, prices are completely flexible; hence, changes in the nominal interest rates reflect changes in the expected inflation rate. In this scenario, a price rise in an exchange rate means a rise in the price of the foreign currency. Therefore, we must see a positive relationship between the exchange rate and the nominal interest rate differential between two countries. A more recent approach based on a Keynesian view (Keynes, 1923; Dornbusch, 1976) assumes a sticky price in the short run. These two different approaches were further extended by Frankel (1979), who developed a model which incorporates assumptions of both traditions. To incorporate this idea, we chose to use as covariates the slopes of the interest rates, defined as the difference between the long term interest rate and the short term interest rate, and the immediate interest rates, which represent the daily cost of money, both for USD and EUR. These are presented in Figure 4 (a) and (b).

Purchasing Power Parity (PPP)

The theory of purchasing power parity (PPP), coined by Gustav Cassel (1918), uses the long-term equilibrium exchange rate between two currencies to equalize their purchasing power in their home country. Thus, the Purchasing Power Parity (PPP) exchange rate is the rate that equates the two currencies by eliminating the differences in the price levels between countries. Although the effects of PPP on long-term exchange rate forecasting have been supported in past literature (Rogoff, 1996), the effects on short-term forecasting have not been supported (Rogoff, 1996; Lothian and Taylor, 1996; Grossman and Rogoff, 1995). The "mean reversion" hypothesis – exchange rates to revert to their PPP values in a longer period – was supported by empirical studies. Xu (2003) suggested the use of aggregate price indices to test the fitness of PPPs or exchange rate forecasting. We choose to use the differential of inflation measured as the difference between the log-changes in the Consumer Price Indices. It is presented in Figure 5.

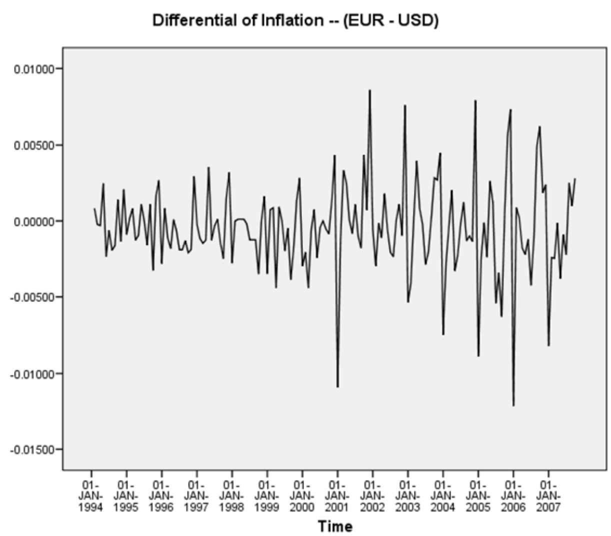

Figure 5. Inflation differential between EMU and United States

Money aggregates

Since the implementation of the flexible exchange rate system in the early 1970s, researchers have become interested in the monetary approach to exchange rate forecasting. In this approach, the relative price of two currencies is influenced by the supply and demand for money in two countries. Money aggregates measure the stock of money in circulation within a country. It is assumed that monetary aggregates are sufficiently sensitive to interest rates. Changes in monetary aggregates are considered to reflect a change in monetary policy, economic outlook, and inflationary pressures. However, it is difficult to know the direction of the causality effect of money aggregates on interest rates, and therefore on exchange rates. There are several ways in which monetary aggregates can be calculated: M1 ("narrow money") includes currency in circulation plus checkable deposits and travelers checks; M2 ("broad money") is essentially M1 plus savings deposits and small (less than $100,000) time deposits; and M3 is the widest measure, consisting of M2 plus large time deposits and a number of other relatively liquid instruments. In this study we use the log-changes in M3 aggregates as covariates. The evolution of the log-changes in M3 is shown in Figure 4(c).

Business Cycles

Business cycles have been considered in the literature as possibly influencing the movement of exchange rates. Since share prices track the business cycle quite closely, we include a measure of business cycles in our analysis by using the log of the share price indices computed by the OECD. These indices are normalized such that their levels in 2000 were 100.

Dataset

The data have been downloaded from the OECD statistical database, available online at http://stats.oecd.org/wbos/default.aspx. The data cover monthly financial and economic indicators for the longest time interval available in the OECD database. Note the special case of the Euro Monetary Union. The starting date of the time series corresponding to that geographic zone often starts before the actual historic inception time of the Euro Monetary Union. In those cases, the calculations were conducted by the OECD statistics units. One of the reasons we kept those data points, aside from the fact that the series would be too small to decently fit ARIMA models if we had not, was that all series come from the same source ensuring coherence in the calculations. More information can be found on the OECD website.

Models and Results

In this section, we explain the tenants and underpinnings of ARIMA models. It should be noted that the methodology of ARIMA estimation and model selection is a classical topic covered in most textbooks on time series analysis (e.g. Brockwell and Davis, 2003; Hamilton, 1994; Tsay, 2005; Wei, 2006). We do not intend to duplicate here the description of already well documented methodologies, but rather to give a practical meaning in this context to the models. We present three models: one with the EUR per USD exchange rate only (i.e. with no covariates), which we call the simple ARIMA model; and two ARIMA models with added covariates, one comprising all of our chosen economic indicators, and another restricting the covariates to a selected few.

The modeling approach

The simple ARIMA model

AutoRegressive Integrated Moving Average (ARIMA) models intend to describe the current behavior of variables in terms of linear relationships with their past values. These models are also called Box-Jenkins (1984) models on the basis of these authors' pioneering work regarding time-series forecasting techniques. An ARIMA model can be decomposed in two parts. First, it has an Integrated (I) component (d), which represents the amount of differencing to be performed on the series to make it stationary. The second component of an ARIMA consists of an ARMA model for the series rendered stationary through differentiation. The ARMA component is further decomposed into AR and MA components. The autoregressive (AR) component captures the correlation between the current value of the time series and some of its past values. For example, AR(1) means that the current observation is correlated with its immediate past value at time *t-1*. The Moving Average (MA) component represents the duration of the influence of a random (unexplained) shock. For example, MA(1) means that a shock on the value of the series at time *t* is correlated with the shock at *t-1*. The Autocorrelation Function (ACF) and Partial Autocorrelation Function (PCF) are used to estimate the values of p and q, using the rules reported in Table 1. In the next section, we provide an example of a simple ARIMA model.

Table 1. Characteristics of the theoretical ACF and PACF for stationary processes (excerpted from Wei, 2005 p.109)

Process	ACF	PACF
AR(*p*)	Tails off as exponential decay or damped sine wave	Cuts off after lag *p*
MA(*q*)	Cuts off after lag *q*	Tails off as exponential decay or damped sine wave
ARMA(*p,q*)	Tails off after lag *(q-p)*	Tails off after lag *(p-q)*

Example of a simple ARIMA model

As we have already noticed when we presented the EUR per USD exchange rate, its time series exhibits some nonstationarity. In fact, if we refer to the guidelines in Table 1, we see that only the ACF and the PACF of the first-differenced USD/EUR series, in Figure 3, feature a decaying pattern with reasonable cut-off points. Conjointly, Table 1 and Figures 2 and 3 suggest an ARIMA(2,1,0) structure. It implies the following evolution equation

$$\left(1-\varphi_1 B-\varphi_2 B^2\right)\left[\left(Y_t-Y_{t-1}\right)-\mu\right]=a_t$$

which can be written

$$\begin{aligned} Y_t &= \mu\left(1-\varphi_1-\varphi_2\right)+\left(1+\varphi_1\right)Y_{t-1} \\ &\quad +\left(\varphi_2-\varphi_1\right)Y_{t-2}-\varphi_2 Y_{t-3}+a_t, \end{aligned}$$

where Y_t represents the exchange rate (in EUR per USD), B is the backshift operator, and a_t is random noise. We estimated this model using the ARIMA procedure in SPSS[1] through a maximum likelihood procedure assuming that no data are missing[2], and reported the results in Table 2.

Table 2. Simple ARIMA(2,1,0) for EUR per USD exchange rate

Model Fit statistics					
Akaike (AIC)	Bayesian (BIC)	RMSE	Ljung-Box Q(18)	DF	Sig.
-839.758	-830.440	0.019	13.936	16	0.603
		Estimates	Std Error	t	≈ Sig.
Coeff.	AR1	0.392	0.077	5.082	0.000**
	AR2	-0.185	0.077	-2.396	0.018**
	Constant	-0.001	0.002	-0.629	0.530

(*) Significant at 10% level (**) Significant at 5% level

The Ljung-Box statistic of the model is not significantly different from 0 with a value of 13.936 for 16 degrees of freedom and an associated p-value of .603, thus failing to reject the null hypothesis of no remaining significant autocorrelation in the residuals of the model. This indicates that the model seems to adequately capture the correlation information in the time series. Moreover, the low root mean square error (RMSE) indicates a good fit for the model. Both AR coefficients, φ_1 and φ_2, are significantly different from 0 with values 0.392 (0.000) and -0.185 (0.018) respectively. The numbers in parentheses are the p-values. This model enables us to write the following evolution equation for the exchange rate,

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[1]The ARIMA procedure in SPSS estimates the following *ARIMA(p,d,q)* model:

$$\varphi_p(B)\left[\Delta\left(Y_t-\sum_{i=1}^{m}c_i X_{it}\right)-\mu\right]=\theta_q(B)a_t$$

where B is the backshift operator, $\Delta=(1-B)^d$ is the differencing operator, $\varphi_p(B)=1-\varphi_1 B-\varphi_2 B^2-\cdots-\varphi_p B^p$ is the AR polynomial, $\theta_q(B)=1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q$ is the MA polynomial, and X_{it} are predictors of the dependent variable Y_t.

[2] This procedure is called Melard's algorithm in SPSS. Another procedure, using the Kalman filter, is available for efficient maximum likelihood estimation in the case of missing data.

$$\hat{Y}_t = 9.2 \times 10^{-4} + 1.392\,Y_{t-1} - 0.578\,Y_{t-2} + 0.185\,Y_{t-3} \quad (1)$$

where $\hat{Y}_t$ represents the value estimated by the model for Y_t. Notice the extremely small value of the constant in this equation, which is reflective of the lack of significance of the estimate $\hat{\mu}$, with a value of -0.001 (0.530). This equation establishes the evolution of the EUR per USD exchange rate as a weighted sum of its past three values plus a random shock. Furthermore, one interesting feature is, that because the time series of the exchange rate needs to be differenced once to be made stationary, the coefficients of this linear combination of past values sums to one. That is, the current value of the exchange rate can be interpreted as a weighted average of its past values. Notice also that the coefficients decrease in absolute value from *t-1* to *t-3*, therefore giving proportionally more weight to the most recent values.

Regression and ARIMA on errors

We have seen above that a simple ARIMA model can provide an evolution equation with a simple interpretation. Nonetheless, this type of model can be criticized because it fails to provide an explanation of the causal structure behind the evolution of the time series. One simple and common way to try to get at the structural relationships behind the phenomenon we model is to use linear regression. Note that it certainly does not amount to proving causal relationships (Pearl, 2000), but it provides a simple and easily interpretable equation that attempt to model relationships. However, a linear regression of the EUR per USD exchange rate on explanatory variables, such as economic indicators, also has some drawbacks. In this section, we briefly expose why regression is not suitable and proceed to explain how the ideas of linear regression and time series modeling can be combined. We support our explanation with examples of two models that combine both ideas. These final results are summarized in the last subsection.

Combining good ideas: from regression to ARIMA on errors

Linear regression is used to estimate linear relationships between a dependent variable and explanatory variables. This relationship can be summarized in an equation of the type $Y_t = \mu + cX_t + \varepsilon_t$. One key assumption in that modeling is that the random components ε_t are independent and identically distributed. In particular, their values should not be correlated. However, when one regresses the exchange rate against the nine covariates chosen above and a time trend, one finds that the Durbin-Watson statistic, which is a measure of the correlation between successive values of the random components, is 0.273. This suggests a strong positive correlation between the random components at times *t-1* and *t*. Moreover, the sample ACF and PACF of the residuals of the regression, displayed in Figure 6 (a) and (b), show that the first 9 autocorrelations of the time series are significant. These results confirm that the linear regression model fails to account for the correlation across time, and furthermore, that the random components of that regression follows an ARIMA model. In fact, the sample ACF and PACF, conjointly with the plot of the time series of the residuals, in Figure 6 (c), suggest that the residuals are stationary and may be modeled by an ARMA(1,0) model.

The idea then is to build a model that combines a regression and an ARMA(1,0) model on the errors. If this is the right intuition, because the coefficients of the regression component and the coefficients of the ARMA(*p*,*q*) component are modeled at the same time, it may turn out that the orders, *p* and *q*, may not be exactly the same as the orders of the ARMA model that would fit the residuals of the first regression[3]. In our case, we found that *p* should be equal to 3, in order to account for the autocorrelation in the residuals. That is, if we write $\varepsilon_t = Y_t - cX_t - \mu$, supposing that the current value of the random shock depends only on the values of the last three random components, we want ε_t to follow an evolution equation of the form $\varepsilon_t - \varphi_1\varepsilon_{t-1} - \varphi_2\varepsilon_{t-2} - \varphi_3\varepsilon_{t-3} = a_t$, where a_t is truly an independent random noise. This translates into an evolution equation for Y_t which is given by

$$\begin{aligned} Y_t &= \mu(1-\varphi_1-\varphi_2-\varphi_3) \\ &\quad + c\left(X_t - \varphi_1 X_{t-1} - \varphi_2 X_{t-2} - \varphi_3 X_{t-3}\right) \\ &\quad + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \varphi_3 Y_{t-3} + a_t. \end{aligned} \quad (2)$$

We see that this last evolution equation for Y_t reflects the dependence of Y_t on its past as well as a dependence on some explanatory variable X_t and the past values of this independent variable. Notice that the dependence of Y_t on its past is summarized by the coefficients $\varphi_1, \varphi_2, \varphi_3$, and that the same coefficients dictate the dependence of Y_t on the past of the explanatory variables, up to a multiplicative constant. In the following section we present two models based on this blueprint.

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[3] Note though that the stationarity of the model should not change. In other words, if the residuals of the regression are stationary and modeled by an ARMA, then it is an ARMA component that will be found in the combined model. Conversely, if the residuals exhibits nonstationarity and can be modeled by an ARIMA(p,d,q), the combined model should have an ARIMA(p',d,q') with *p*' and *q*' possibly different from *p* and *q*.

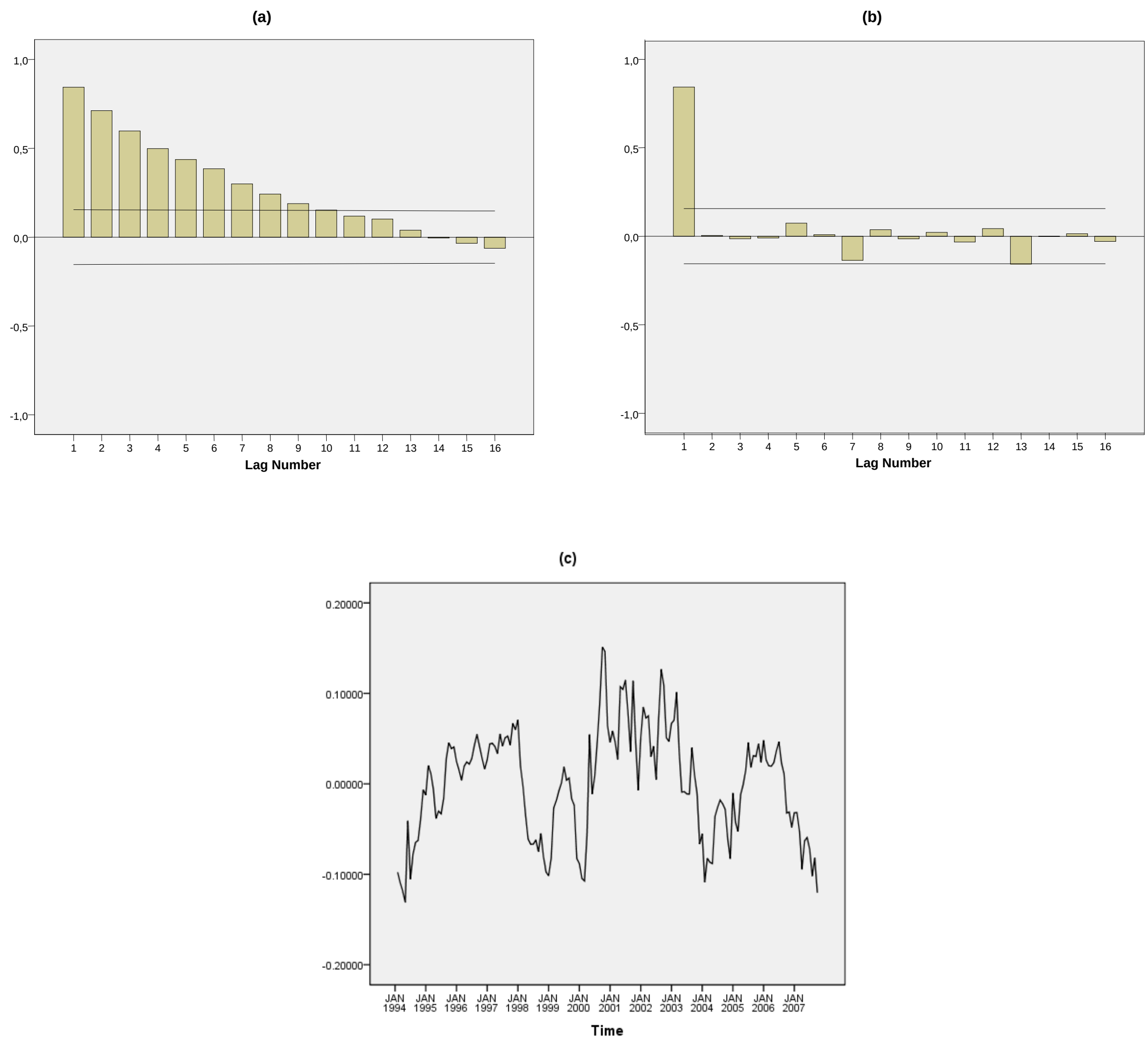

Figure 6. Sample ACF and PACF of the unstandardized residuals of the regression of EUR per USD on the EUR interest rate gradient, US interest rate gradient, EUR immediate interest rate, USD immediate interest rate, differential of inflation, log-variations of M3 aggregates for EUR and USD, and log-levels of the share price indices for EMU and USA: (a) ACF of residuals; (b) PACF of residuals; (c) time series of the unstandardized residuals of the regression.

Results

We used the ARIMA procedure in SPSS to estimate two models using the ideas exposed above. We report the results in tables 3 and 4. Both models impose an ARMA(3,0) structure on the errors of a regressions. Thus, only their regression components distinguish between them. The first, in table 3, models the EUR per USD exchange rate in terms of its past and the past and present values of 9 covariates: the interest rate differentials in the EMU and the USA, the immediate interest rates on EUR and USD, the inflation differential between EMU and USA, the log-variations of M3 aggregates for EUR and USD, and the log-levels of the share price indices for EMU and USA.

The first two things to note concern the performance of the model in terms of fit to the EUR per USD time series. First, we notice that this model, with a Ljung-Box statistic Q(16) of 9.385 for a p-value of 0.897, captures the autocorrelation structure of the EUR per USD time series well. Second, compared to the simple ARIMA(2,1,0) model presented in Table 2, this model exhibits a improved fit with lower values for the Akaike Information Criterion (AIC) (-855.45 vs. -841.37) and the root mean square error (0.017 vs. 0.019). However, the Bayesian Information Criterion (BIC), another statistic for model selection, like the AIC, is significantly higher for the ARIMA(2,1,0) model (-815.07 vs. -

Table 3. ARMA(3,0) model of EUR per USD exchange rate, with nine covariates

Model Fit statistics					
Akaike (AIC)	Bayesian (BIC)	RMSE	Ljung-Box Q(18)	DF	Sig.
-855.451	-815.073	0.017	9.385	16	0.897

	Estimates	Std Error	t	≈ Sig.
Coefficients:				
AR1	1.398	0.081	17.218	0.000**
AR2	-0.621	0.132	-4.696	0.000**
AR3	0.205	0.081	2.549	0.012**
EUR IR	-0.004	0.006	-0.677	0.500
USD IR	-0.003	0.009	-0.285	0.776
EUR differential	0.005	0.008	0.587	0.558
USD differential	-0.001	0.007	-0.197	0.844
LogM3- EUR	1.122	0.656	1.711	0.089*
LogM3- USD	-0.354	0.308	-1.152	0.251
DifInf	0.288	0.247	1.169	0.244
s[t] EMU	0.285	0.052	5.461	0.000**
s[t] USA	-0.299	0.064	-4.691	0.000**
Constant	1.025	0.195	5.269	0.000**

Notes: (IR: Immediate Interest rate; differential: differential of the Interest rates; LogM3: Ln(M3[t]) - Ln(M3[t-1]) ; DifInf : Differential of Inflation, s[t] : ln(Shares[t])) (*) Significant at 10% level (**) Significant at 5% level

835.16). This suggests that the improvement in terms of fit for the more complex model has come at the cost of a loss of parsimony. Indeed, many of the covariates' regression coefficients are not significantly different from zero, with the exception of the share prices indices both for the EMU and the USA (significant at a 5% level), and a marginally significant coefficient for the log-variation of the European M3 aggregate. Notice also that the constant and the AR coefficients are all significant at a 10% level.

In order to improve this latter model, we estimated a second model, keeping as covariates only the share price indices. Again, we find that this new model captures well the dependence across time of the values of the EUR per USD exchange rate, with a Ljung-Box statistics Q(16) of 8.47 and a p-value of 0.935. The constant and the AR coefficients, $\mu, \varphi_1, \varphi_2$ and φ_3, are again all significantly different from 0, with values 0.955(0.000), 1.375(0.000), -0.616(0.000) and 0.225(0.004). This model however exhibits an improvement on all fit statistics – AIC, BIC and RMSE—when compared to both the ARIMA(2,1,0) model and the ARMA(2,0) model with all possible covariates. Moreover, all the regression coefficients are significantly different from 0, with 0.290(0.000) and -0.292(0.000) for the EMU and the USA respectively.

One could write the evolution equation given by the latter model simply by noticing that equation 2 given in the section above is easily generalizable to multiple explanatory variables and then replacing each coefficient by its estimated value in Table 4. However, it is more interesting to notice that the regression coefficients for the share price indices are very close in absolute value, and in fact, considering the standard errors of each estimate, not distinguishable. We therefore decided to set both equal in absolute value to 0.290. Denoting by s_t^{EMU} and s_t^{USA} the logs of the share price indices, we can then write the evolution equation of the EUR per USD exchange rate, denoted Y_t, in terms of the current and past values of the difference in levels between the share indices in each currency and the past values of Y_t, as follows:

$$\begin{aligned} Y_t &= 0.016 + 0.290\left(s_t^{EMU} - s_t^{USA}\right) - 0.398\left(s_{t-1}^{EMU} - s_{t-1}^{USA}\right) \\ &+ 0.179\left(s_{t-2}^{EMU} - s_{t-2}^{USA}\right) - 0.065\left(s_{t-3}^{EMU} - s_{t-3}^{USA}\right) \\ &+ 1.375\,Y_{t-1} - 0.616\,Y_{t-2} + 0.225\,Y_{t-3} + a_t. \end{aligned} \tag{3}$$

Table 4. ARMA(3,0) model of EUR per USD exchange rate, with two covariates

Model Fit statistics					
Akaike (AIC)	Bayesian (BIC)	RMSE	Ljung-Box Q(18)	DF	Sig.
-869.141	-850.469	0.017	8.417	16	0.935

	Estimates	Std Error	t	≈ Sig.
Coefficients:				
AR1	1.375	0.077	17.900	0.000**
AR2	-0.616	0.123	-4.994	0.000**
AR3	0.225	0.077	2.936	0.004**
s[t] EMU	0.290	0.049	5.966	0.000**
s[t] USA	-0.292	0.061	-4.751	0.000**
Constant	0.955	0.183	5.205	0.000**

Notes: (**) Significant at 5% level

Discussion

While it is not our purpose to claim new results in the field of exchange rate modeling and forecasting, it is nonetheless interesting to discuss and compare the models we have found in a slightly broader context than models fit and selection. In this section, we first discuss the models we found together, before expanding the scope of the discussion by including the critiques that have been made in the literature on exchange rates modeling of econometric and especially ARMA models.

We focus our comparison of our models to the models described by equations 1 and 3. Perhaps the first thing to notice when comparing equations 1 and 3 is the closeness of the coefficients in front of the lags of Y_t. This actually explains the puzzling fact that one is an ARMA(3,0) model while the other is an ARIMA(2,1,0) model. Remember that our first ARIMA model imposed that the sum of the coefficients be one (since $1+\varphi_1$, $\varphi_2 - \varphi_1$ and $-\varphi_2$ add up to 1). In the estimation of an ARMA model, for constraints of stationarity, the sum of the coefficients is constrained to be less than 1. However, in our ARMA(3,0) model, the sum is close to 1 with a value of 0.984. Thus, this model exhibits features close to those of an integrated model. Second, the insight on the dynamic of Y_t gained by introducing the logs of the share price indices over the ARIMA(2,1,0) model is not negligible. Indeed, equation 3 states that the share price indices influence the EUR per USD exchange rate through their log differences. If the logs are equal, then equation 3 reduces to an evolution equation very close to equation 1. Conversely, if we assume the difference between the logs of each index to be constant over a period of three months, then the cumulative effect of this difference on the EUR per USD rate, i.e. the euro price of a US dollar, will be 0.285 times the difference in the log levels of the share indices.

Several researchers have claimed that the Random Walk (RW) model outperforms econometric and time series techniques (Fernandes, 1998; Kilian and Taylor, 1991) for the forecasting of exchange rates. In particular, the performance of the latter in the short run is found to be poor (Goodman, 1979; Meese and Rogoff, 1983; Wolff, 1988; Diebold and Nason, 1990; Chinn and Meese, 1995). For example, Meese and Rogoff (1990) found that the RW model was superior to an ARMA model. Moreover, it is argued that the strict assumptions required by an econometric model and time series models such as ARIMA sometimes may not be suitable to capture nonlinearity. Hence, other researchers have proposed a hybrid approach that combines both parametric methods and non-parametric methods for input selection (Ince and Trafalis, 2006).

Our ARIMA models do not necessary contradict the market efficiency hypothesis for the following reasons. First, all the models exhibit an integration feature with regard to the levels of exchange rates, since the sum of the coefficients is close to 1. Thus, we can safely say that the time series of the exchange rates in fact exhibit characteristics close to RWs. Indeed, the point in saying that a process is a random walk is to stress on the unpredictability of the increments. To see that our models do not necessarily contradict this unpredictable feature, it is worth considering equation 2. In equation 2, using the fact that the sum of the AR coefficients is close to one, we see that unless the levels of the covariates have changed significantly over the course of four months, their contribution is in fact very small, and the model is then comparable to the ARIMA(2,1,0) of equation 1, again. Second, the market efficiency hypothesis states that at any given time all the information available is reflected in the price of the goods in the market, here the relative price of currencies as compared to the USD. That is to say that at any given time t, the information available cannot help in predicting the prices at $t+1$, and the best predictor is the level of prices at t. In other words, the best forecasting model is the RW. However, if information becomes available between time t and $t+1$, it will be incorporated in the price, and therefore, the movements of the price depend on other variables. In our case, however, we are studying a posteriori the structure of the exchange rates, using extended ARIMA modeling. Indeed, we are explaining, not predicting, the movement of the prices, and it is therefore easy, and sensible, to find better models than a RW, which by definition explains nothing.

We may actually argue that if we were to test the forecasting efficiency of our models then, since they have been built to explain the past, they will be biased toward the past in the sense that they will weigh past information more heavily than new information, and will perform poorly. To our knowledge, and our understanding, this is a salient problem in the modeling paradigm that is not easily solved. We can relate this point back to the market efficiency hypothesis, and the available information. On the FX markets, information is incorporated extremely fast, faster than a monthly or even a weekly frequency. So models using weekly (or lower frequency) data do not allow one to quantify how markets react to some information, such as a change in interest rates, because the change has already occurred and the information has been consumed by the time you predict it. Following this line of thought, we can safely say that it is very probable that no matter how good our models are to explain the movements of the exchange rate, the best predictive model may still be a random walk.

From a macroeconomic point of view, it is worth noting that none of the economic indicators predicted by the theory put forth in the first part of this paper has turned to be supported by the empirical facts. The exercise in the pages above is but a demonstration of the difficulty of finding models that can do more than accurately forecast after the fact! While the models used here are sensible, they fall short of succeeding as true forecasting models.

We have provided an interesting example of the use of ARIMA models to reproduce and explain a time series. We have been mindful of explaining the assumptions and underpinnings of the general models, as well as relating the estimated models to their practical implications. We trust that this material will serve as a useful case study for teachers and students of macroeconomics.

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