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Hidden Markov Model for the detection of a degraded operating mode of optronic equipment

Titre: Modèle de Markov caché pour la détection d'un mode de fonctionnement dégradé d'un équipement optronique

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Abstract: As part of optimizing the reliability, Thales Optronics now includes systems that examine the state of its equipment. The aim of this paper is to use hidden Markov Model to detect as soon as possible a change of state of optronic equipment in order to propose maintenance before failure. For this, we carefully observe the dynamic of a variable called "cool down time" and noted Tmf, which reflects the state of the cooling system. Indeed, the Tmf is an observation of the hidden state of the system. This one is modelled by a Markov chain and the Tmf is a noisy function of it. Thanks to filtering equations, we obtain results on the probability that an appliance is in degraded state at time t, knowing the history of the Tmf until this moment. We have evaluated the numerical behavior of our approach on simulated data. Then we have applied this methodology on our real data and we have checked that the results are consistent with the reality. This method can be implemented in a HUMS (Health and Usage Monitoring System). This simple example of HUMS would allow the Thales Optronics Company to improve its maintenance system. This company will be able to recall appliances which are estimated to be in degraded state and do not control too early those estimated in stable state.

Résumé : Dans le cadre de l'optimisation de la fiabilité, Thales Optronique intègre désormais des systèmes d'observation de l'état de santé de ses équipements. Dans cet article nous utilisons des chaînes de Markov cachées pour détecter le plus tôt possible le changement d'état d'un équipement optronique afin de proposer une maintenance avant la panne. Pour cela, nous observons attentivement la dynamique d'une variable appelée "temps de mise en froid" et notée Tmf, qui reflète l'état du système de mise à froid. En effet, le temps de mise en froid est une observation de l'état caché de notre système. Ce dernier est modélisé par une chaîne de Markov et le Tmf est une fonction bruitée de cette chaîne. Grâce à des équations de filtrage, nous avons obtenu des résultats concernant la probabilité que l'équipement soit dans un état dégradé à l'instant t, connaissant l'histoire des Tmf jusqu'à cet instant. Nous avons ensuite évalué numériquement l'approche proposée sur des données simulées. Puis, pour finir nous avons appliqué notre méthodologie afin d'analyser nos données réelles et nous avons pu vérifier la cohérence des résultats obtenus. Cette méthode peut être implémentée dans le HUMS (Health and Usage Monitoring System). Cet exemple simple de HUMS pourrait permettre à Thales Optronique d'améliorer son système de maintenance. L'entreprise sera en mesure de définir un protocole de maintenance conditionnelle à l'état estimé du système.

Keywords: Chaîne de Markov cachée, Filtrage, Détection de rupture
Mots-clés : Hidden Markov Model, Filtering, Breakage detection
AMS 2000 subject classifications: 35L05, 35L70

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1. Introduction

Thales Optronics aims to optimize the ratio availability - cost. The company wants to reduce the failure rate of these appliances by the evolution of its maintenance concept which passes from a logic of repair to a logic of anticipation of these defects. As part of optimizing the reliability, Thales Optronics now includes systems that examine the state of its equipment. This function is performed by HUMS (Health and Usage Monitoring System). The role of HUMS is :

1. to record environmental conditions and use of equipment,
2. to evaluate the state of the system,
3. to anticipate and alert about the excesses of operation,
4. to optimize maintenance operations.

Our approach comes within a specific context. In this paper, we focus on point 2. We have at our disposal a variable that reflects the state of the system and we want to detect a change in mode of this variable (which is a change of slope in our case). There exist different methods for this kind of detection as the CUSUM, presented for instance by Basseville and Nikiforov (1993). But in this paper we focus on hidden Markov chains to detect this change of mode. The state of our system at time t is then modeled by a Markov chain X_t. In our case we do not observe directly this chain but indirectly through the Tmf variable, a noisy function of this chain. We will see in this paper how we can address this issue by using filtering theory.

For this, we will first introduce the industrial problem in section 2 and the mathematical model in a general case in section 3. Section 4 presents a simulation study and section 5 the implementation of the methodology on our real data.

2. Industrial problem

Each of the appliances has a logbook which provides the following information at each start-up: number of uses, cumulative operating time of appliance, initial temperature and the "cool down time" (Tmf for "temps de mise en froid" in french). This Tmf is the transit time for the system from ambient temperature to a very low one. This temperature decrease is required to operate appliance and this is done on every boot. According to experts, a Tmf increase results from deterioration in the system. According to this hypothesis, a careful observation of Tmf evolution would allow us to determine the state of the system and prevent the breakdown. So we will look at the evolution of Tmf which seems to be a good indicator of the system state.

We suppose that the system has three possible states:

– Stable state: Tmf is constant. This reflects a system in good working order. There is no anomaly to report.
– Degraded state: the Tmf increases. This reflects a specific deterioration in the system.
– Failure: the system is stopped.

Appliances move from stable state to degraded state, to reach failure. It is important to detect the beginning of a degradation to prevent as soon as possible occurrence of failure. As explained above, we would not observe directly the state of our system but indirectly through the Tmf. Our objectives are:

– to estimate at every moment the state of the system by the evaluation of the probability of being in degraded state knowing the history of Tmf until this moment,
– to detect as soon as possible the degradation of the system for a maintenance action before failure.

To solve this problem, we use hidden Markov chains.

3. Hidden Markov model

In this section, we provide a general mathematical framework to tackle our problem. We have to detect a rupture in the behavior of the variable Tmf. There exist different methods for this kind of detection (see for instance Basseville and Nikiforov, 1993). We choose to use Hidden Markov Model (HMM). HMM are frequently used to detect point mutations in DNA in genomics (see for instance J.Fridlyand et al., 2004) or in speech recognition (see L.R.Rabiner, 1989). In the domain of reliability, HMM are also used in a context of high frequencies data (see Wang et al., 2004). Moreover, Vrignat et al. (2012) show that this model can be a decision support, which allows maintenance manager to control the degradation level of a process and to have a background Experience "off line" about maintenance activities impact. In our context, the size of our data is not large (28 appliances, maximum 400 recordings in a logbook). But in the following, we will see that this tool is also powerful in our context. We first present the model in a general case and the estimation of the parameters of interest.

3.1. Modeling

3.1.1. Main process

Consider $(X_t)_{t>0}$ a Markov chain in continuous time, defined on a probability space (Ω, F, P) with discrete state space S=$\{e_1, e_2, \dots, e_N\} \subset \mathbb{R}^N$ et de matrice de transition P_t. So $X_t = (X_t^1, \dots, X_t^N)$ is a vector of $\mathbb{R}^N$. For convenience, we follow assumptions from Elliott et al. (1995, p.182) and we set $e_i = (0, 0, \dots, 1, 0, \dots, 0)$ so that $(e_1, e_2, \dots, e_N)$ is an orthonormal basis of $\mathbb{R}^N$.

Let us denote the probability $p_t^i = P(X_t = e_i)$ for $1 \leq i \leq N$ and $p_t = (p_t^1, \dots, p_t^N)$. The motion of the chain X_t depends on $A = (a_{ij})$, the Q-matrix of the process (see Cocozza-Thivent, 1997, p.247/248 for definition) defined by:

$$a_{ij} = \lim_{h \to 0+} \frac{P_h(i,j) - P_0(i,j)}{h},$$

where the $P_h(i,j) = P(X_{t+h} = j | X_t = i)$ are the terms of the transition matrix. If $i \neq j$, a_{ij} represents the intensity of jump from e_i to e_j. The vector p_t is linked to matrix A by the following equation: $\frac{dp_t}{dt} = A^T p_t$ (see Cocozza-Thivent, 1997, p.251). The process X_t has the semimartingale representation (see Cocozza-Thivent, 1997, p.269):

$$X_t = X_0 + \int_0^t AX_r dr + V_t \tag{1}$$

with V_t a martingale.

3.1.2. Observation process

X_t is not directly observed, but through the process Y_t given by the formula:

$$Y_t = Y_0 + \int_0^t c(X_r)dr + W_t \tag{2}$$

with:

– $(W_t)_{t>0}$ a standard Brownian motion on (Ω, F, P) independent of $(X_t)_{t>0}$,
– $c(X_t) =< X_t; c >$ where $<;>$ is the scalar product in $\mathbb{R}^N$ and $c = (c_1, \dots, c_N) \in \mathbb{R}^N$. This represents the slope of Y_t according to the state of X_t. Indeed, if $X_s = e_i$ for $s \in [t - \Delta_t, t]$, $c(X_s) = c_i$ and $Y_t - Y_{t-\Delta_t} = c_i\Delta_t + (W_t - W_{t-\Delta_t})$.

So, in mean, the increase of the observed process Y_t depends on the state of X_t through $c(X_t)$. A Brownian noise is added to the slope $c(X_t)$.

Let us denote:

– $(\mathscr{Y}_t)_{t\geq 0}$ the right-continuous complete filtration generated by $\sigma(Y_s : 0 \leq s \leq t)$,
– $(\mathscr{G}_t)_{t>0}$ the right-continuous complete filtration generated by $\sigma(X_s, Y_s : 0 \leq s \leq t)$.

Recall that our aim is to determine the probability of the system to be in a particular state knowing the trajectories of Y until t. The best L^2-approximation of this quantity is given by the conditional probability $\hat{p}_t^i = P(X_t = e_i \,|\mathscr{Y}_t)$ for $1 \leq i \leq N$. Note that with our choice for e_i,

$$P(X_t = e_i \,|\mathscr{Y}_t) = P(X_t^i = 1 \,|\mathscr{Y}_t) = E\left[X_t^i|\mathscr{Y}_t\right] = (E\left[X_t|\mathscr{Y}_t\right])_i \,.$$

So we have to compute the N-dimensional conditional expectation $E\left[X_t|\mathscr{Y}_t\right]$. This is the aim of the next section.

3.2. Filtering equations and parameters estimation

First, we give filtering equations which provide conditional expectations of functions of X_t, knowing the history of Y_t. Then we will see how these equations allow us to estimate parameters A, c and the probability of being in a state e_i given $\mathscr{Y}_t$.

3.2.1. Filtering equations

Elliott et al. (1995, p189-194) gives unnormalized filtering equations of different fonctionnais of X_t. To write these equations, let us denote $\sigma(F(X_s, s \leq t)) = \bar{E}[\bar{\Lambda}_t F(X_s, s \leq t)\,|\mathscr{Y}_t]$ with $\bar{P}$ and $\bar{\Lambda}_t$ associated with the absolutely continuous probability of change:

$$\left.\frac{dP}{d\bar{P}}\right|_{\mathscr{G}_t} = \bar{\Lambda}_t = \exp\left(\int_0^t < c; X_r > dY_r - \frac{1}{2}\int_0^t < c; X_r >^2 dr\right).$$

This change of probability is a standard method in filtering because under $\bar{P}$, Y_t is independent of X_t. Under $\bar{P}$, the dynamic of unnormalized filter satisfies stochastic differential equation.

Filtering equations are about:

– state of the system:

$$\sigma(X_t) = \sigma(X_0) + \int_0^t A\sigma(X_r)dr + \int_0^t diag(c)\sigma(X_r)dY_r,$$

– number of jumps from e_i to e_j in the time interval $[0,t]$ denoted ς_t^{ij}:

$$\sigma(\varsigma_t^{ij}X_t) = \int_0^t < \sigma(X_r); e_i > a_{ji}e_i dr + \int_0^t A\sigma(\varsigma_r^{ij}X_r)dr + \int_0^t diag(c)\sigma(\varsigma_r^{ij}X_r)dY_r, \quad (3)$$

– waiting time in state e_i on the interval $[0,t]$ denoted ϑ_t^i:

$$\sigma(\vartheta_t^i X_t) = \int_0^t < \sigma(X_r); e_i > e_i dr + \int_0^t A\sigma(\vartheta_r^i X_r)dr + \int_0^t diag(c)\sigma(\vartheta_r^i X_r)dY_r, \quad (4)$$

We can also have informations about the slope thanks to filtering equation on the drift, defined by $T_t^i = \int_0^t < X_r; e_i > dY_r$:

$$\sigma(T_t^i X_t) = c_i \int_0^t \sigma(< X_r, e_i > e_i)dr + \int_0^t A\sigma(T_t^i X_r)dr + \int_0^t \left[< \sigma(X_r); e_i > e_i + diag(c)\sigma(T_r^i X_r)\right] dY_r.$$

These equations about ς^{ij}, ϑ^i, T^i are useful for the estimation of A and c when Y_t is observed in a long time. Note that all these equations are stochastic differential equations so that their solutions can be approximated thanks to Euler scheme approximation (see for instance Kloeden et al., 1992, Part 10.2).

3.2.2. *Estimation and prediction*

Using filtering equations, it is possible to compute the estimated probability of the system to be in state e_i thanks to the following formula:

$$E[X_t | \mathscr{Y}_t] = \frac{\sigma(X_t)}{\sigma(1)}. \quad (5)$$

Indeed, $P(X_t = e_i | \mathscr{Y}_t) = (E[X_t | \mathscr{Y}_t])_i = (\frac{\sigma(X_t)}{\sigma(1)})_i$. However, we need to estimate parameters A and c because $\sigma(X_t)$ depends on them. So by maximum likelihood we estimated A and c, from observations of Y_s on interval $[0,t]$. Estimators are given by the following formulas:

$$\hat{a}_{ij}(t) = \frac{\sigma(\varsigma_t^{ij})}{\sigma(\vartheta_t^i)}, \quad (6)$$

$$\hat{c}_i(t) = \frac{\sigma(T_t^i)}{\sigma(\vartheta_t^i)}. \quad (7)$$

These estimators converge in probability when t goes to infinity according to Elliott et al. (1995, p.188). However, all the parameters are intricaded in each of the filters $\sigma(\varsigma_t^{ij}X_t), \sigma(\vartheta_t^i X_t)$ and $\sigma(T_t^i X_t)$ so that we need an EM algorithm to estimate them. They are fixed points of this EM

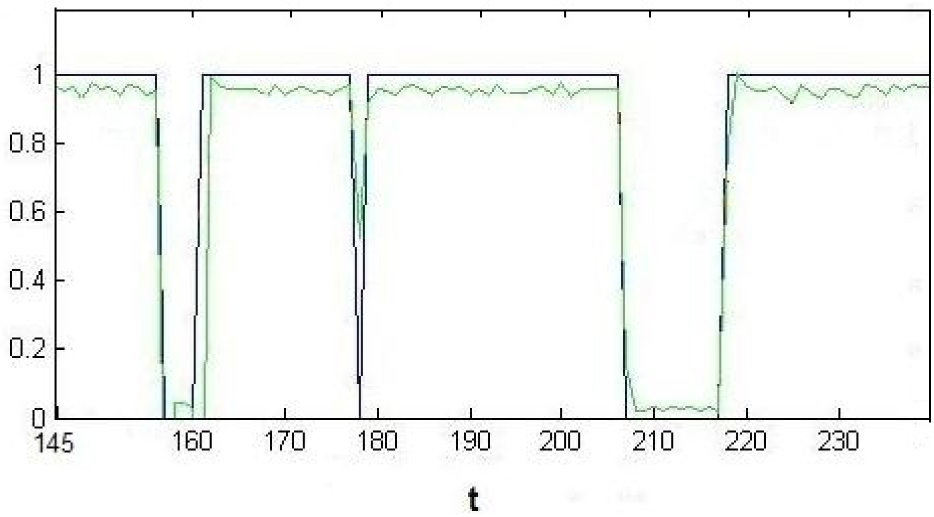

FIGURE 1. *Exemple of path of* X_t^2 *(black line) and its estimation* $P\left(X_t^2 = 1\,|\mathscr{Y}_t\right)$ *(green line)*

algorithm whose principle is developed in Section 4.2.1. This convergence of the sequence of parameters is discussed in Zeitouni and Dembo (1988).

Note that filtering equations do not give directly $\sigma(\varsigma_t^{ij})$, $\sigma(T_t^i)$, $\sigma(\vartheta_t^i)$ and $\sigma(1)$ but $\sigma(\varsigma_t^{ij}X_t)$, $\sigma(T_t^iX_t)$, $\sigma(\vartheta_t^iX_t)$ and $\sigma(X_t)$. To pass from one to another, we just have to multiply these elements by vector $(1,1,..,1)^T$ to obtain $\sigma(\varsigma_t^{ij})$, $\sigma(T_t^i)$, $\sigma(\vartheta_t^i)$ and $\sigma(1)$. Indeed, thanks to assumptions on $X_t \in (e_1,\ldots,e_N)$, $\langle X_t;(1,1,..,1)\rangle = 1$.

Let us now illustrate this approach on simulated data.

4. Simulation study

In this section, the framework is the following. We assume that the process has two possible states: $X_t = e_1$ and $X_t = e_2$ with transitions from e_1 to e_2 and conversely. So when $X_t = e_1$ (respectively $X_t = e_2$), its first coordinate $X_t^1 = 1$ and its second coordinate $X_t^2 = 0$ (respectively $X_t^1 = 0$ and $X_t^2 = 1$). Here, e_1 (respectively e_2) corresponds to the stable state (respectively the degraded state) and X_t oscillates between these two states. In our case, we are more interested in visits to the degraded states which are related to the second hidden state ($X_t = e_2$).

4.1. Probability estimation of being in a degraded state

We first suppose that we know matrix A and vector c. The component a_{12} of A is the parameter of the exponential distribution of the waiting time in stable state (before degraded state) and a_{21} is the parameter of the exponential distribution of the waiting time in degraded state. Using these values, we can simulate X_t. Then, using values of c and X_t, we simulate Y_t thanks to equation (2) and Euler scheme approximation (see for instance Kloeden et al., 1992, Part 10.2) to simulate stochastic differential equations. Now that our data are simulated and our parameters known we can compute the conditional probability that the system is in degraded state. For this, we use equation (5) for the computation of the conditional probabilities $\hat{p}_t^2$ and $\hat{p}_t^1 = 1 - \hat{p}_t^2$. This computation is made again by a recursive algorithm that uses the Euler scheme to approximate stochastic differential equations.

An illustration of the good numerical behavior of the computational process is given in Figure 1. This figure zooms on a part (arbitrary chosen) of the trajectory of X_t^2 and $\hat{p}_t^2 = E\left[X_t^2 | \mathscr{Y}_t\right]$. We clearly observe that the filter correctly provides the evaluation of X_t^2 that is close to 1 (respectively 0) when $X_t^2 = 1$ (respectively when $X_t^2 = 0$). This part arbitrary chosen is an example of the behavior of the trajectory and the other parts of the trajectory show the same behavior.

Note that in this simulation study, we assume that parameters A and c are known. This is not the case in practice and these parameters must be estimated before estimating probability $\hat{p}_t^1$ and $\hat{p}_t^2$.

4.2. Parameters estimation

4.2.1. Estimation of matrix A and vector c

With simulations of process Y_t in a long time, it is possible to use formulas (6) and (7) to estimate parameters A and c.

We first suppose vector c known and we seek to estimate the matrix A from observations of Y_s for $s \in [0,t]$. However, one difficulty of this estimation step is the fact that $\sigma(\varsigma_t^{ij})$ and $\sigma(\vartheta_t^i)$ from equation (6) are governed by A. So we developed an iterative algorithm to approximate A starting with an arbitrary A_0, operating in the following way: at step k, we use $\hat{A}_{k-1}$ and the history of Y_t to approximate (thanks to Euler scheme approximation - Kloeden et al., 1992, Part. 10.2) $\sigma(\varsigma_t^{ij})$ and $\sigma(\vartheta_t^i)$ given by filtering equations (3) and (4) multiplied by the vector $(1,1,..,1)^T$. From these two approximated elements, we compute $\hat{A}_k$ via formula (6). Our convergence criterion of this EM algorithm is a difference between the estimated value and the true value less than 10^{-2} for a number of steps fixed at 1100. The convergence of this estimator has been proved by Zeitouni and Dembo (1988).

Now we assume matrix A known and we seek to estimate vector c from observations of Y_t. For this, we also use formula (7). Once again, $\sigma(T_t^i)$ and $\sigma(\vartheta_t^i)$ are governed by c itself. So, by the same method as previously, we developed again an iterative algorithm to approximate c starting with an arbitrary vector c_0.

4.2.2. Sensitivity of filter $P(X_t = e_2 | \mathscr{Y}_t)$ to parameters A and c

Since the values of A and c are unknown in practice, it seems important to study the impact of a poor estimation of A and c in computation of probability $P(X_t = e_2 | \mathscr{Y}_t)$. We simulate X_t and Y_t for given values of A and c: $A = \begin{pmatrix} -0.1 & 0.1 \\ 0.05 & -0.05 \end{pmatrix}$ and $c = (-1;1)$. We then estimate probability of being in a degraded state.

In a first step, these two different matrices: $A_1 = \begin{pmatrix} -0.01 & 0.01 \\ 0.04 & -0.04 \end{pmatrix}$ and $A_2 = \begin{pmatrix} -0.2 & 0.2 \\ 0.08 & -0.08 \end{pmatrix}$ are considered, instead of the true matrix A. With these two matrices, we again compute probability of being in a degraded state. Figure 2 gives these estimations. We clearly observe that wrong matrices A_1 and A_2 do not severely impact on the estimate of probability of interest.

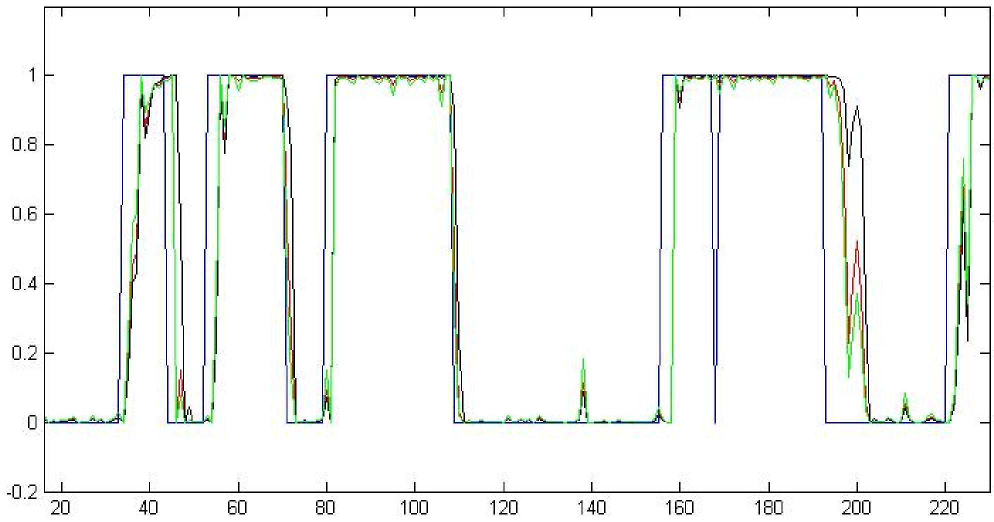

FIGURE 2. *Evolution of* X_t^2 *(blue line) and its estimation* $P\left(X_t^2 = 1 \,|\, \mathscr{Y}_t\right)$ *for the true A (red line) and different matrices A:* A_1 *(black line),* A_2 *(green line)*

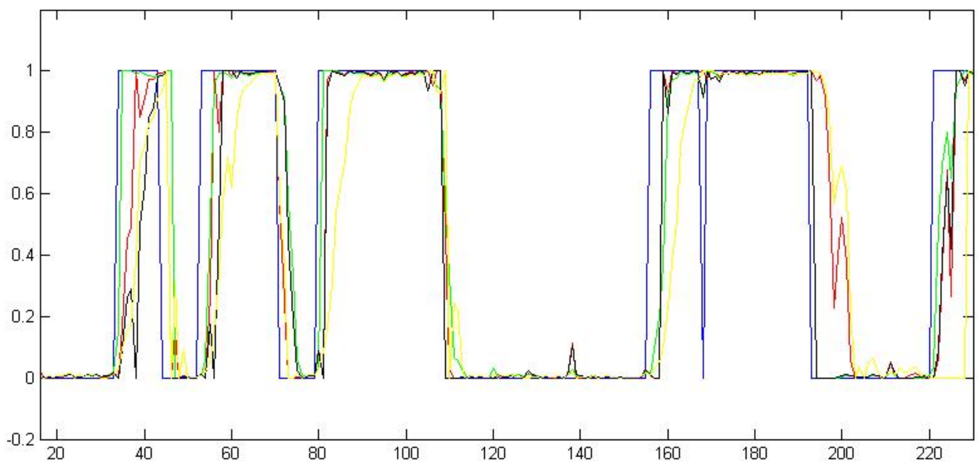

FIGURE 3. *Evolution of* X_t^2 *(blue line) and its estimation* $P\left(X_t^2 = 1 \,|\, \mathscr{Y}_t\right)$ *for the true c (red line) and different vectors c:* c_1*(green line),* c_2 *(black line) and* c_3 *(yellow line)*

In a second step, we estimate this probability with values $c_1 = (-0.5, 0.5)$, $c_2 = (-1, 0.5)$ et $c_3 = (0, 1)$ in place of the true value c. Figure 3 gives these corresponding estimations. Again we observe that deviations do not severely impact the probability of interest.

5. Application to industrial case

5.1. Data

We have logbooks of 28 appliances: five of them failed due to a mechanical malfunction in the cooling system. For other appliances, the failures were not mechanical and are considered to be unpredictable (not related to a degradation effect and often due to an electronic failure). From the logbooks, we recover Tmf value and initial temperature at each startup of the system. The time unit of the model is the number of startups and we assume the same model for all appliances (A and c are the same for all of them) and the motion of the 28 appliances are mutually independent.

5.2. Preliminary data processing

The two variables, Tmf and initial temperature are linked together. Indeed, a high (resp. low) initial temperature increases (resp. decreases) the Tmf. So it was necessary to correct this crude Tmf by a standard linear regression according to initial temperature of appliance. We use this regression to bring the Tmf to a setting where initial temperature is constant and equals 10°C. This corrected Tmf is denoted by Tmf_r in the following. In Figure 4, we provide the corrected Tmf evolution of one appliance. We can see a very noisy phenomenon. Down peaks may be the result of "on/off/on" too brutal for appliance: the system is on, turned off and back on instantly so that initial temperature remains low. To soften this phenomenon, we decide to smooth the corrected Tmf (Tmf_r). For this, we compute a moving average of Tmf_r as follows:

$$Tmf_l(j) = \frac{\sum_{i=j}^{20+j-1} Tmf_r(i)}{20},$$

where $Tmf_r(i)$ is the value of corrected Tmf at the i^{th} startup. Let us denote by Tmf_l the smoothing correcting Tmf value. In our modeling, we set $Y_t = Tmf_l(t)$. A theoretical interest of the smoothing step is that the filtering method works well with a not too noisy signal. Indeed, we noticed empirically that when the data are not smoothed, the estimation of the probability of being in a degraded state presents very big jumps sometimes with an amplitude equal to one. Note that the Tmf_l starts at the 20^{th} startup because it is necessary to have 20 Tmf_r to compute Tmf_l. In practice it is necessary to wait 20 startups before the first computation of the probability of being in a degraded state. In Figure 4, we plot the evolution of smooth corrected Tmf of this appliance.

We can notice on the bottom graph of Figure 4 that Tmf_l remains constant for a while and then gradually increases. This change in slope was not obvious in the top graph of Figure 4. This is another interesting point of the smoothing part. Now, from these Tmf_l values, we are able to compute probability of being in degraded state. For this, we first need to estimate parameters A and c.

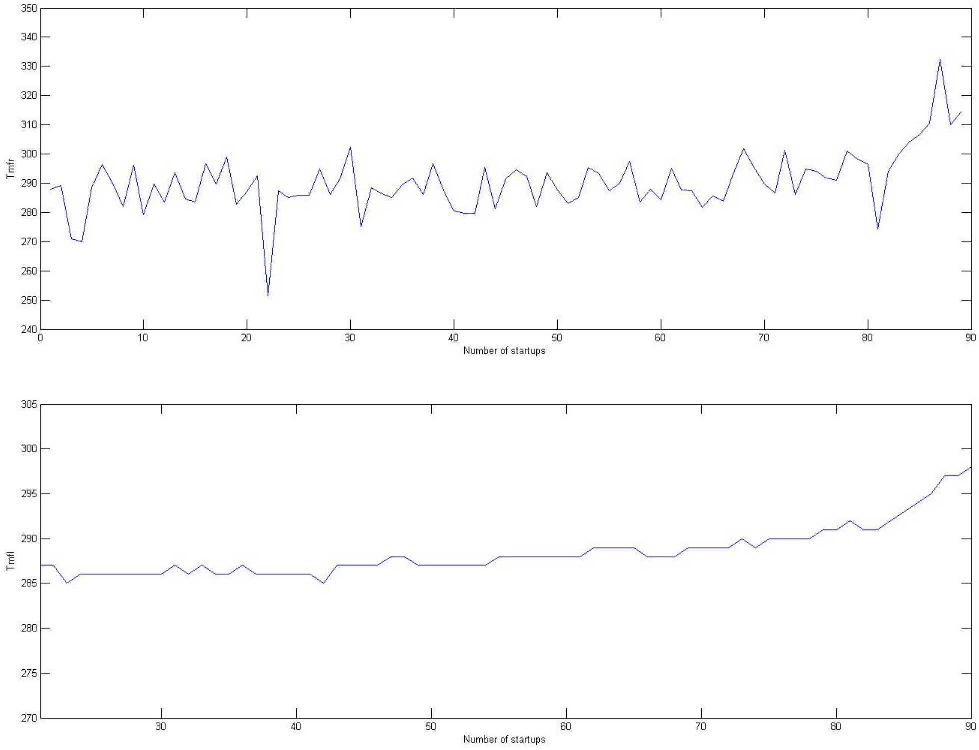

FIGURE 4. *Evolution of corrected Tmf over uses at 10°C and associated smoothed data for one appliance*

5.3. Estimation of parameters A and c

The estimation method presented in section 4.2.1 using the observation of the process in a long time is not possible here. Indeed the real system does not oscillate between two states because it is stopped at its first transition to degraded state. Then we propose a practical choice for c and A mixing estimation using the data from the 28 appliances and expert opinions. Note that from simulations in section 4.2.2, we have noticed that a misspecification of these parameters does not seem to strongly impact on the filter value $P(X_t = e_2 | \mathscr{Y}_t)$.

According to experts, slope of smoothed curve is close to 0 when the system is in stable state and it is strictly positive when it degrades. In addition, according to graphs of the evolution of the Tmf_l, the slope is close to 1 when the system is in a degraded state. So we can naturally set $\hat{c} = (0, 1)$.

About the Q-matrix $A = \begin{pmatrix} -a_{12} & a_{12} \\ a_{21} & -a_{21} \end{pmatrix}$, a_{12} is the parameter of the exponential distribution of the time in stable state (before degraded state). We have estimated this parameter using our data (28 appliances: 5 times of failure and 23 censures). By standard survival method taking censures into account, we have first estimated a_{12} by $\frac{1}{1000}$. In order to detect as soon as possible a change of state (contraint requested by Thales) and according to our study of sensitivity (see Section 4.2.2), we chose to put a value 10 times greater that is $\hat{a}_{12} = \frac{1}{100}$.

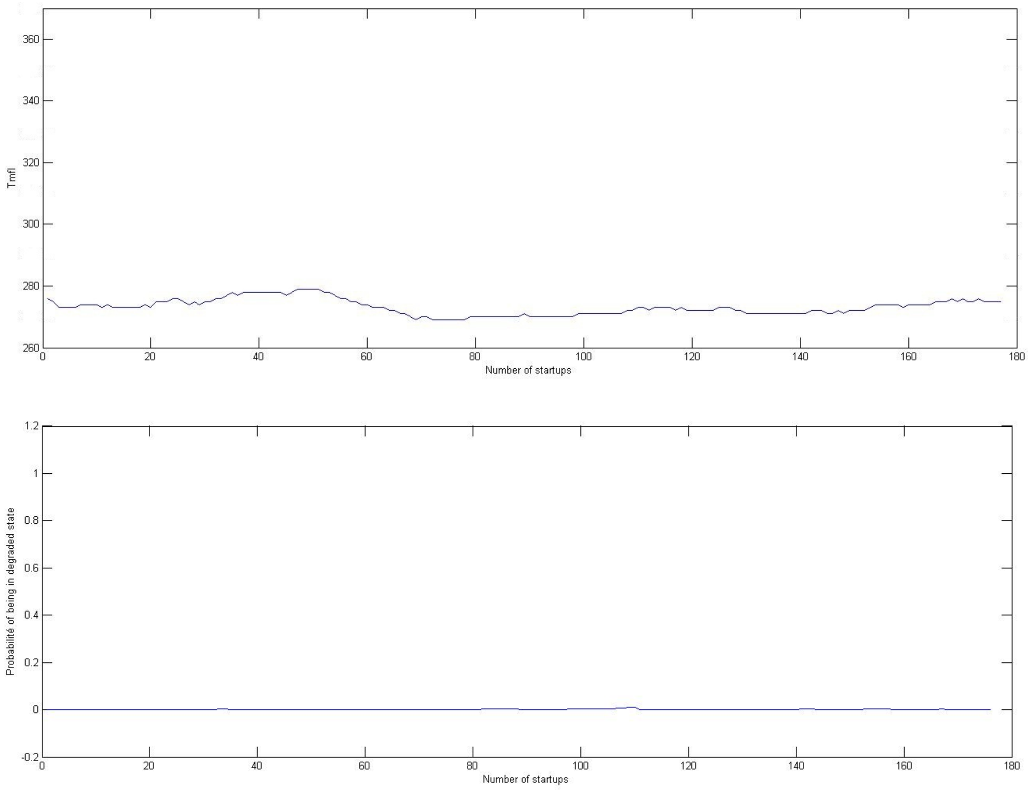

FIGURE 5. *Smoothed Tmf_r for one appliance E_h and evolution of the corresponding probability to be in a degraded state*

The coefficient a_{21} should equal zero because system in degraded state can not return to a stable state. But in our equations, our filter $P(X_t = e_2 | \mathscr{Y}_t)$ must be versatile, so we have chosen a small value $\hat{a}_{21} = \frac{1}{1000}$. With this choice, the chance that an appliance in degraded state comes to stable state is very small.

Now, we are able to estimate probability of interest.

5.4. Results

With this choice for A and c and using the filtering equation (5), we computed the probability of being in a given state, at each startup t, knowing the history $\mathscr{Y}_t$. We first consider appliance noted E_h. A posteriori, we can see that E_h was trouble-free during its whole history. Figure 5 gives the evolution of its Tmf_l. At each time t, we estimate its probability of being in degraded state through (5) using values of Tmf_l before t. We clearly observe a Tmf_l quite constant during uses and a probability of being in a degraded state close to zero.

Now, we consider an appliance denoted E_d. A posteriori, we see that E_d degrades and breaks down. In Figure 6, we see a Tmf_l quite constant during the first uses; then, Tmf_l increases and then decreases to return to starting level. In fact, this is due to a deterioration of the ball bearing

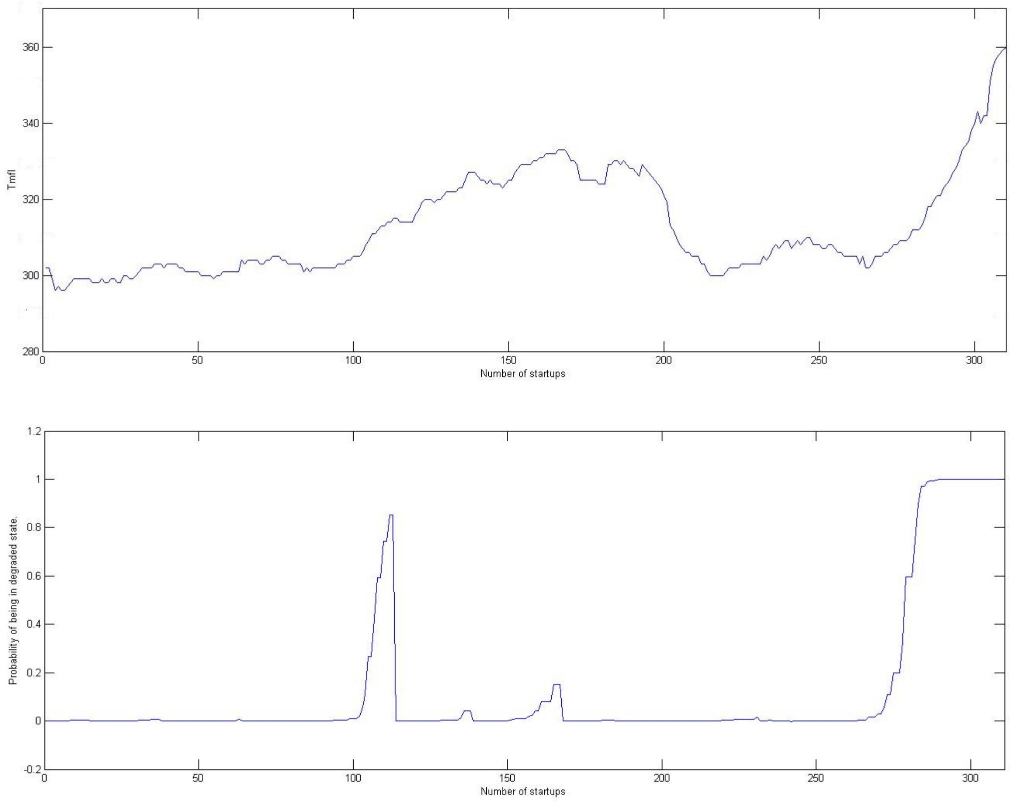

FIGURE 6. *Smoothed Tmf_r for one appliance E_d and evolution of the corresponding probability to be in a degraded state*

system. This deterioration leads to an increase of Tmf_l but this has not an influence on the cooling system. The ball bearing system corrects itself (and leads to a decrease of Tmf_l). Finally, we notice an abrupt rise of Tmf_l. Simultaneously, we note that the computed probability of being in degraded state is very low when Tmf_l is constant and then sharply increases with Tmf_l to one.

To conclude, these two examples illustrate a good numerical behavior of the proposed approach. Now we are interested in a decision criterion that allows us to detect as soon as possible a degraded state in order to return appliances to perform maintenance action before failure.

6. Decision criterion

The increase of the probability of being in a degraded state is not sufficient to detect a future failure. We have to propose a decision criterion for maintenance. For this, we have tested different rules based on the fact that probability has to cross a threshold during a number of consecutive uses. We have tried different thresholds combined with different numbers of crossing. At each time, we have recorded the number of false and good detections. It is important to limit both false-positive and false-negative detections. According to the comparison of these rules, we have chosen the following criterion: when the probability of being in a degraded state equals 1 over a

TABLE 1. *Results obtained with the decision criterion*

Decision criterion	Observed failure	No observed failure
Future failure detected	**3**	0
Future failure not detected	2	**23**
Total	5	23

period of three uses, the appliance is sent back for maintenance. We applied this rule on our 28 appliances and we obtain the results presented in Table 1.

The decision criterion provides 26 good detections over 28. It does not provide false detection: the 23 appliances without observed failure were not detected as degraded. For the five appliances that failed, the criterion identifies three of them as degraded (before failure). It doesn't seem that the wrong identification is related to the prior knowledge on A and c. Nevertheless, let us also note that we have shown in Section 4.2.2 that the probability of detection was not very sensible to an error on the parameters. Rather, we suppose that for the two appliances which have not been correctly identified, the failure may be not related to a degradation effect of the cooling system and then can not be detected by our proposed approach.

Note that some appliances have Tmf increases due to a malfunction of the system of ball bearings that is able to repair itself and our criterion is chosen in order to avoid such types of detection. To detect the two undetected failures, the threshold would have to be greatly lowered to 0.3 and in this case there would be many false detections.

7. Concluding remarks

Thanks to filtering equations, we obtained results on the probability that an appliance is in degraded state at time t, knowing the history of the Tmf until this moment. We have evaluated the numerical behavior of our approach on simulated data. Then we have applied this methodology on our real data and we have checked that the results are consistent with the reality.

Using this model, Thales is now working to implement in its HUMS in operating system a new maintenance algorithm.

There are two technical solutions according to the system embedded calculator:

- system capitalizes data, assesses and provides information about the cooler state,
- system capitalizes data but the cooler state is assessed by a maintenance laptop plugged periodically on its maintenance socket.

This model will allow us to improve the maintenance and the usage policies of monitored system. The improvements are:

- moving from a preventive or corrective maintenance to a predictive maintenance, this evolution allows to reduce the support cost,
- ability to create a degraded operational mode,
- increase the mission success probability (systems will be chosen according to their real status for critical mission).

The performance of the new maintenance policy is possible thanks to combination of mathematical, high technology and new maintenance organization.

References

Basseville, M. and Nikiforov, I. (1993). *Detection of Abrupt Changes: Theory and Application*. Prentice-Hall.

Cocozza-Thivent, C. (1997). *Processus stochastiques et fiabilité des systèmes*. Edition Springer.

Elliott, R., Aggoun, L., and Moore, J. B. (1995). *Hidden Markov Models*. Edition Springer.

J.Fridlyand, Snijders, A. M., Pinkel, D., Albertson, D., and l, A. N. J. (2004). Hidden markov models approach to the analysis of array cgh data. *Journal of Multivariate Analysis*, 90:132–153.

Kloeden, P., E.Platen, and H.Schurz (1992). *Numerical solution of SDE*. Edition Springer-Verlag.

L.R.Rabiner (1989). A tutorial on hidden markov models and selected applications in speech recognition. *Proceedings of the IEEE*, 77:257–285.

Vrignat, P., Avila, M., F.Duculty, B.Robles, S.Begot, and F.Kratz (2012). Indicators generation as part of a policy of conditional preventive maintenance. *Congrès de Maîtrise des Risques et Sûreté de Fonctionnement.*$\lambda\mu$*18*, 34:890–893.

Wang, W., Guan, X., and X.Zhang (2004). Modeling program behaviors by hidden markov models for intrusion detection. *The Third IEEE International Conference on Machine Learning and Cybernetics (ICMLC' 2004), Shanghai, China*, 77:2830–2835.

Zeitouni, O. and Dembo, A. (1988). Exact filters for the estimation of the number of transitions of finite-state continuous time markov processes. *IEEE Transactions on Information Theory*, 34:890–893.