

Extreme Rainfall Analysis at Ungauged Sites in the South of France : Comparison of Three Approaches

Titre: Analyse des pluies extrêmes en des sites non-jaugés dans le sud de la France : Comparaison de trois approches

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Abstract: We compare three approaches to estimate the distribution of extreme rainfall at ungauged sites. Two approaches rely on the univariate generalized extreme value distribution (GEV). SIGEV interpolates linearly the GEV parameters estimated locally. RFA is a regional method which builds circular homogeneous neighborhood around each site in order to increase the sample size. The observations in the neighborhood, properly normalized, are assumed to follow the same GEV distribution. Then the normalizing factor (called the index value) has to be interpolated to ungauged sites. The third method is the stochastic hourly rainfall generator called SHYPRE. By characterizing precisely rainfall events, SHYPRE is able to simulate long rainfall series with statistics similar to the observed series. The distribution of extreme rainfall is estimated empirically from the simulated series. The three approaches are evaluated and compared on datasets from over 1000 rain gauges in the South of France. The evaluation framework that we follow is based on the computation of high-level quantiles and aim at assessing the goodness-of-fit of the three approaches and their sensitivity to the training data. Our conclusions are threefold : SIGEV, as implemented, should be avoided because of its lack of robustness, RFA and SHYPRE despite the fact that they are based on very different hypotheses on rainfall provide comparable performance and finally, the main challenge regarding the estimation at ungauged sites concerns the spatial interpolation of the parameters, whatever the approach taken.

Résumé : Nous comparons trois approches pour l'estimation de la distribution des pluies extrêmes en des sites non-jaugés. Deux de ces approches reposent sur la loi des valeurs extrêmes généralisée (GEV). La méthode SIGEV interpole linéairement les paramètres de la GEV estimés localement aux sites jaugés. RFA est une méthode régionale qui définit des voisinages homogènes circulaires autour de chaque site ce qui permet d'augmenter la taille de l'échantillon. En effet, RFA fait l'hypothèse que les observations aux stations du voisinage suivent la même loi GEV à un facteur de normalisation près. Ce facteur, appelé *index value* doit être interpolé aux sites non-jaugés. La troisième approche se base sur un générateur de pluie horaire appelé SHYPRE. À l'aide d'une caractérisation précise des événements pluvieux, SHYPRE est en mesure de simuler de longues séries de pluie ayant des statistiques semblables aux séries d'observations. Ces trois approches sont évaluées et comparées sur un jeu de données comprenant plus de 1000 stations dans le sud de la France. La comparaison des méthodes repose sur le calcul de quantiles de haut niveau et a pour but d'évaluer la justesse et la sensibilité des méthodes. Nos conclusions sont les suivantes : SIGEV tel que mis en oeuvre ne devrait pas être retenu en raison de son manque de robustesse, RFA et SHYPRE ont des performances comparables bien que ces méthodes soient très différentes et nous concluons que le défi le plus important à relever pour ces trois approches réside dans l'interpolation spatiale des paramètres.

Keywords: Generalized Extreme Value distribution, Regional Frequency Analysis, Stochastic hourly rainfall generator

Mots-clés : Loi des valeurs extrêmes généralisée, Analyse régionale fréquentielle, générateur stochastique de pluie horaire

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1. Introduction

The South of France includes three mountain ranges : the Pyrenees in the West along the Spanish border with a peak at 3 404 m, the *Massif central*, a smooth mountain range which lies at the North of the Mediterranean coast and peaks at 1 885 m and the Alps, a great mountain range at the border with Switzerland and Italy with many peaks higher than 4 000 m. The region is subject, on one hand, to the Atlantic oceanic climate from the West and, on the other hand, to the Mediterranean climate from the South. The complex orography together with the combination of oceanic and Mediterranean influences explain the high variability of rainfall in the region and the occurrence of intense rainfall events which may trigger floods and landslide with dramatic human and material consequences (Delrieu et al. (2005)). Hence, the analysis of extreme rainfall is a critical step in the design of civil engineering structures (e.g. dams, reservoirs and bridges) or for urban and landscape planning.

Extreme value theory (EVT – Coles (2001)) provides the proper parametric framework to model the distribution of extremes. The Generalized Extreme Value (GEV) distribution is often employed to model annual maxima of rainfall at gauged sites. The GEV parameter estimates can then be interpolated in order to obtain a model for extreme rainfall at ungauged sites. Several interpolation schemes have been proposed such as smoothing with kernel techniques (Carreau and Girard (2011); Daouia et al. (2011); Gardes and Girard (2010)) and artificial neural network and kriging (Ceresetti et al. (2012)). The natural extension of the univariate GEV distribution to spatial maxima are the max-stable processes (Padoan et al. (2010)) which allow accurate modeling of the dependence structure of spatial maxima.

The so-called regional frequency analysis (RFA) approaches consider, in addition to local observations, observations from other sites in a region. Within this extended pool of observations, extremes are assumed to be identically distributed apart from a site-specific scaling factor. This assumption is referred to as regional homogeneity and can be validated by statistical tests (Hosking and Wallis (1997); Viglione et al. (2007)). A potential advantage of RFA is to improve the accuracy of the estimated distribution by reducing the uncertainty associated with extreme observations (Kysely et al. (2011)). Moreover, RFA can be applied for the estimation of the distribution of extreme rainfall at ungauged sites. This requires the definition of a homogeneous region for each site and the estimation of site-specific factors. The cornerstone of regional frequency analysis is the approach pioneered by Hosking and Wallis (1997). Fixed homogeneous regions defined a partition of the sites. Another variant of the RFA approach is to define *regions-of-influence* for each site (Burn (1990); Castellarin et al. (2001)). These regions, which can be thought of as neighborhoods, are based on similarity of sites attributes such as geographic and climatological characteristics.

An alternative approach to the analysis of extremes is by means of stochastic rainfall generators. Hidden Markov Models were proposed to model the occurrence and intensity of rainfall Hughes et al. (1999); Charles et al. (1999), resampling based on nearest neighbours Buishand and Brandsma (2001) and conditional mixtures with artificial neural networks Carreau and Vrac (2011) among others. These models aim at modelling the distribution of rainfall at a given time-step (daily or hourly) with proper statistical models. Some models might rely on atmospheric information as covariates and can be used to study the impact of climate change on precipitation. Other types of rainfall generators rely on the event-based nature of the precipitation process.

Poisson cluster models simulate storms from which originate raincells with various lifetimes and rainfall intensities [Burton et al. \(2008\)](#). Arnaud and Lavabre [Arnaud and Lavabre \(1999\)](#) describe a stochastic hourly rainfall generator which is based on the description of rainfall events both at the hourly and daily time-step. Rainfall generators can simulate long series of observations from which various statistics of rainfall can be estimated empirically.

In this paper, our goal is to compare the estimation of extreme rainfall distribution in terms of high-level quantiles at sites which were kept apart for validation. For this reason, methods based on univariate distributions are adequate, there is no need at this stage to model the spatial dependence structure. We consider three approaches. In the first approach, local GEV parameter estimates are spatially interpolated to ungauged sites thanks to geographic covariates. Preliminary experiments with non-parametric non-linear interpolation showed no clear improvements over simple linear interpolation. We thus kept the linear interpolation of the local GEV parameter estimates, called the SIGEV method, for further comparisons. Second, we consider a regional frequency approach with a circular region-of-influence type of neighborhood. It is straightforward to choose the size of the neighborhood given by the radius by iteratively testing for homogeneity. The GEV is taken as the model of the scaled regional sample distribution. The third approach is a regional version of the rainfall generator from [Arnaud and Lavabre \(1999\)](#) which is presented in section 3. In all three approaches, some model parameters need to be spatially interpolated to be applicable at ungauged sites. Since the spatial interpolation bears similarities in all cases, it is described in section 4. In section 5, we present the rainfall data and the evaluation framework applied in the paper. In section 6, the comparative results are summarized. Finally, we provide a discussion and a conclusion in section 7.

2. Methods based on extreme value theory

Let X_i be the daily annual maximum rainfall at station i , $i = 1, \dots, n$. The RFA and the SIGEV methods make the assumption that X_i follows a Generalized Extreme Value (GEV) distribution, which we write $X_i \sim \mathcal{G}\mathcal{E}\mathcal{V}(\mu_i, \sigma_i, \xi_i)$, whose distribution function is given by :

$$F_i(x) = \exp \left[- \left\{ 1 + \xi_i \left(\frac{x - \mu_i}{\sigma_i} \right) \right\}^{-1/\xi_i} \right], \quad \text{for } 1 + \xi_i \left(\frac{x - \mu_i}{\sigma_i} \right) > 0 \quad (1)$$

where μ_i and σ_i are the location and scale parameters respectively and ξ_i is the shape parameter (or tail index) which controls the heaviness of the upper tail of the distribution. When $\xi_i > 0$, the tail is heavy (Pareto-type tail), the tail is finite for $\xi_i < 0$ and exponential if $\xi_i = 0$ (in this case, the distribution function is obtained by taking the limit of Eq.(1) when $\xi_i \rightarrow 0$).

The choice of the GEV to model annual maxima is justified by the Fréchet-Tippet Theorem which states that the maximum of most random variables over a large number of repetitions converges to a GEV distribution [Coles \(2001\)](#). In practice, we assume that maximum rainfall over a year is independent and identically distributed and that one year is long enough to ensure that the GEV is a good approximation to the distribution of annual maxima. This is a standard assumption. A T -year return level, the level of daily rainfall which is expected to be exceeded on average once every T years, corresponds to the quantile of probability $1 - 1/T$ which, with the

GEV assumption, is given by, for $\xi_i \neq 0$:

$$q_i(T) = \mu_i - \frac{\sigma_i}{\xi_i} \left[1 - \left\{ -\log \left(1 - \frac{1}{T} \right) \right\}^{-\xi_i} \right]. \quad (2)$$

2.1. Spatial interpolation of local GEV parameters

For the SIGEV method, we further assume that the GEV parameters can be interpolated in space thanks to geographic covariates. More precisely, we make the following additional assumptions regarding the GEV parameters from Eq.(1) :

$$\begin{aligned} \mu_i &= \mu(z_i) \\ \sigma_i &= \sigma(z_i) \\ \xi_i &= \xi(z_i) \end{aligned} \quad (3)$$

where z_i is a vector of covariates at station i and $\mu(\cdot)$, $\sigma(\cdot)$ and $\xi(\cdot)$ are functions which spatially interpolate the GEV parameters. The implementation of these functions is detailed in section 4. In practice, the GEV parameters must be estimated locally at each station i and then the interpolator functions are trained on the pairs $(z_i, \hat{\mu}_i^L)$, $(z_i, \hat{\sigma}_i^L)$ and $(z_i, \hat{\xi}_i^L)$, $i = 1, \dots, n$ where $\hat{\mu}_i^L$, $\hat{\sigma}_i^L$ and $\hat{\xi}_i^L$ are the local GEV estimates for site i . In this work, the local GEV parameters are estimated with the L-Moments method Hosking and Wallis (1997). We screened out stations with less than 20 years of observations in order to avoid high variance local GEV parameter estimates. To obtain the GEV parameters at a new site i^* , we must collect the covariates for this site in z_{i^*} and then apply the trained interpolator functions on z_{i^*} . Return levels can then be computed thanks to Eq.(2).

The SIGEV method is rather straightforward to implement. However bias and variance may arise from the difficulty to accurately estimate and interpolate the GEV parameters. In particular, the shape parameter estimate can have a large variance because few observations provide information on the shape of the upper tail of the distribution. This is especially true for stations with small numbers of observations. In addition, the spatial interpolation step has its own bias and variance which interact with those of the local GEV parameter estimates. Besides, the interpolation procedure does not take into account the possible dependencies between the GEV parameters at a given site.

2.2. Regional Frequency Analysis

This approach attempts to solve some of the issues of the SIGEV method. First of all, the shape parameter is assumed to be shared across all sites from a properly defined region. This increases the sample size and reduces the variance of the GEV parameter estimates. Second, only one parameter, called the index value, needs to be interpolated. The RFA method proceeds as follows. Let us define the index value parameter at site i as $m_i = E[X_i]$ and let Y_i be the annual maximum normalized by the index value :

$$Y_i = \frac{X_i}{m_i}. \quad (4)$$

Therefore, we have the following moment equalities :

$$\begin{aligned}
 E[Y_i] &= 1 \\
 \text{Var}[Y_i] &= \frac{\text{Var}[X_i]}{m_i^2} = \text{CV}(X_i)^2 \\
 E[Y_i^3] &= \frac{E[X_i^3]}{m_i^3} = \frac{\gamma(X_i) \text{Var}[X_i]^{3/2}}{m_i^3} = \gamma(X_i) \text{CV}(X_i)^3
 \end{aligned}
 \tag{5}$$

where $\text{CV}(X_i)$ and $\gamma(X_i)$ are the coefficients of variation and of skewness respectively. A homogeneous region is defined as a set of sites $\{i_j; j \in 1 \dots J\}$ such that the following equalities in distribution (indicated by $\stackrel{d}{=}$) hold :

$$Y_{i_1} \stackrel{d}{=} Y_{i_2} \stackrel{d}{=} \dots \stackrel{d}{=} Y_{i_J}
 \tag{6}$$

According to the moment equations in Eq.(5) and assuming that the first three moments are sufficient to describe the underlying distribution, the equalities in distribution are fulfilled whenever the coefficients of variation $\text{CV}(X_{i_j})$ and skewness $\gamma(X_{i_j})$ of the sites $\{i_j; j \in 1 \dots J\}$ are equal. Next, we assume that the set of normalized annual maxima in the homogeneous region is distributed according to a regional GEV distribution :

$$Y_{i_j} \sim \mathcal{G}\mathcal{E}\mathcal{V}(\mu^R, \sigma^R, \xi^R).
 \tag{7}$$

Thus all the observations from the sites in the homogeneous region serve to estimate the regional GEV parameters. Thereby the sample size is increased and the variance of the regional GEV estimates is reduced compared to the local GEV estimates which resort only on the observations from one site. As a consequence of Eqs. (4) and (7), the distribution of the un-normalized annual maxima is given as :

$$X_{i_j} \sim \mathcal{G}\mathcal{E}\mathcal{V}(m_{i_j} \mu^R, m_{i_j} \sigma^R, \xi^R).
 \tag{8}$$

Therefore, all the sites in the homogeneous region share the same shape parameter ξ^R and their location and scale parameters differ by a multiplicative factor given by their index value parameters.

2.2.1. Choice of homogeneous neighborhood

In practice, to apply the RFA approach to a site i^* , we define the neighborhood by a region of influence around the site i^* (the so-called RoI approach [Burn \(1990\)](#)). The region of influence lies in the horizontal plane and is bounded by a circle centered on the site i^* . [Fig.1](#) represents a 40 km circular neighborhood around the city of Montpellier (red dot), in the South of France. The stations which contribute to the regional GEV estimation are marked as blue dots. Since our definition of neighborhood varies with the site i^* , the regional GEV parameters now depend on i^* , so we write $\mu_{i^*}^R$, $\sigma_{i^*}^R$ and $\xi_{i^*}^R$. The size of the circular neighborhood is allowed to change from one site to another in order to fulfill the homogeneity assumption. We optimize the radius for each site as follows :

1. An initial radius is fixed to 50 km, beyond which it is not reasonable to assume homogeneity.
2. Two homogeneity tests (Anderson-Darling [Viglione et al. \(2007\)](#) and Hosking and Wallis [Hosking and Wallis \(1997\)](#)) are carried out to check whether the homogeneity assumption is fulfilled.
3. If one of the test fails, the radius is decreased by 5 km and we start over at step 2.
4. If both tests succeed, we conclude that the region is homogeneous and keep the current radius to define the neighborhood.

Two additional criteria to stop the loop in the optimization of the size of the neighborhood are established in order to prevent bad neighborhood solutions (the radius has to be greater or equal to 5 km and the neighborhood must contain a least 50 observations).

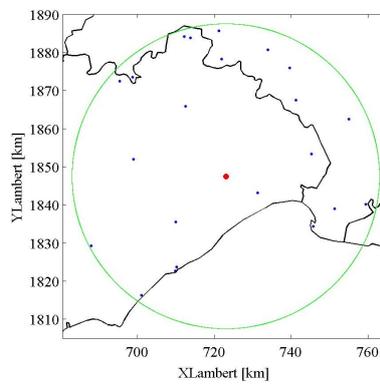


Figure 1: 40 km circular neighborhood around Montpellier, France, (red dot) among the stations from the training set (blue dots).

2.2.2. Estimation at ungauged sites

The first step is to define the proper circular neighborhood around the site i^* of interest as described above. Then, the regional GEV parameters $\mu_{i^*}^R$, $\sigma_{i^*}^R$, $\xi_{i^*}^R$ are estimated from the normalized observations in the circular neighborhood with the L-moments method. To normalize the observations at each site in the neighborhood, the index value parameters are estimated locally by the sample average of the observed daily annual maxima. In addition, in order to fulfill the independence assumption of the observations in the neighborhood, we perform a spatial declustering. More precisely, whenever more than one maximum occurs on a given day among the stations from the neighborhood, we assume that these maxima are associated to the same meteorological event and we keep only the largest maximum. Note that if the site i^* is ungauged, the neighborhood can nevertheless be defined from the gauged sites in the training set and the estimation of the regional GEV parameters is performed with the observations from the gauged sites in the neighborhood.

The only parameter which has to be interpolated at ungauged sites is the index value parameter. This is performed similarly to the interpolation in the SIGEV approach. A vector of geographic covariates z_{i^*} is available for each station i^* and we assume the following relationship :

$$m_{i^*} = m(z_{i^*}), \quad (9)$$

where the m_{i^*} are approximated by the sample average of observed annual maxima to train the interpolator function $m(\cdot)$ whose implementation is postponed to section 4. We screened out stations with less than 10 years of observations to avoid high variance local index value estimates.

The RFA approach thus requires the estimation of four parameters to obtain the GEV parameters at a new site i^* , namely $\mu_{i^*}^R$, $\sigma_{i^*}^R$, $\xi_{i^*}^R$ and $m_{i^*} = m(z_{i^*})$. The return levels can then be computed by applying Eq.(2) with $\mu_{i^*} = m_{i^*} \mu_{i^*}^R$, $\sigma_{i^*} = m_{i^*} \sigma_{i^*}^R$ and $\xi_{i^*} = \xi_{i^*}^R$.

3. SHYPRE : stochastic hourly rainfall generator

SHYPRE (Simulated HYdrographs for flood PRobability Estimation) has been developed by the hydrological research team at the Aix-en-Provence branch of the National Research Institute of Science and Technology for Environment and Agriculture (Irstea) for about two decades. SHYPRE is a framework for hydro-meteorological risk estimation based on a stochastic hourly rainfall generation model linked with a conceptual hydrological model [Cernesson et al. \(1996\)](#); [Arnaud and Lavabre \(1999\)](#).

3.1. At-site rainfall generator

We describe here the stochastic hourly rainfall generator of SHYPRE for gauged sites. For a given rain gauge station, this generator simulates rainfall events at an hourly time-step. The calibration of the generator proceeds by first defining rainfall events from daily rainfall according to the following criteria. Rainfall during the event must exceed 4 mm per day and must exceed 20 mm at least on one day during the event (see Fig. 2 top panel for an illustration). Next, we consider hourly rainfall. A rainy period is defined as a succession of hours with positive rainfall and a shower is a succession of positive hourly rainfall with a unique maximum (in practice, showers are defined by a large hourly intensity surrounded by lower hourly intensities). Note that several rainy periods may coexist within a daily rainfall event and several rainfall showers may coexist within a given rainy period, see Fig. 2 lower panel.

3.1.1. Variables describing hourly rainfall

Nine descriptive variables are used to characterize the structure of hourly rainfall events. The first variable is the number of rainfall events per year, where the rainfall events are defined as described above on daily rainfall. We next consider variables related to hourly rainfall. A given rainfall event consists in several rainy periods (see Fig. 2 lower panel) which are separated by one or several hours with no rain. Two other variables are the number of rainy periods per rainfall event and the duration in hours of the period with no rain between two rainy periods. The following variables are connected with showers. These are the number of showers per rainy period, the duration of a shower, the cumulated rainfall of a shower, the ratio of the maximum hourly rainfall to the cumulated rainfall of a shower and the relative position of the hourly maximum within a shower. The last variable is the number of major showers in a given event. At least one major shower is

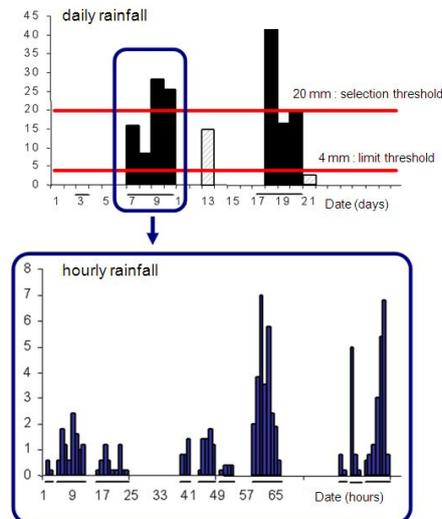


Figure 2: (Top) Two rainfall events as defined on daily rainfall (minimum rainfall of 4 mm and at least one day with 20 mm); (Bottom) Rainy periods within a given rainfall event (shown in blue) defined on hourly rainfall. Rainy periods decompose into showers which are centred around local maxima.

assumed to occur in an event and we consider an additional major shower for each day in the event with cumulated rainfall above 50 mm.

Each descriptive variable is modeled with a parametric distribution (either Gaussian, Exponential, Uniform, Geometric or Poisson, see [Cernesson et al. \(1996\)](#)). The parameters of each distribution are estimated by the method of moments from hourly rainfall observations. Standard practice is to split the year into two seasons and thus there are 20 parameters to estimate for each season. The SHYPRE rainfall generator makes the assumption that the variables describing the rainfall events are independent and that the rainfall process is stationary in time. However, the temporal dependence between successive rainy periods within a rainfall event may be taken into account [Cantet \(2009\)](#).

3.1.2. Density estimation of daily rainfall

The descriptive variables are sampled independently in a precise order dictated by the pattern of rainfall events. From the values sampled, an hourly rainfall series is generated. The rainfall generator is validated by comparing the simulated series with the observed ones. The comparisons are made in terms of maximum rainfall of various durations (1, 2, 3, 4, 6, 12, 24, 48 and 72 hours). These statistics are not part of the nine variables taken into account to describe the hourly rainfall series. The estimation of the generator parameters and the validation of the simulation was carried out on hourly rainfall from 251 pluviographic rain gauges across France, Réunion (Indian Ocean) and Martinique (Caribbean Sea). Thus, the SHYPRE generator proved to be successful in various climates [Arnaud et al. \(2007\)](#). Thanks to the parametrization in terms of rainfall events, rainy periods and showers, the generator is directly applicable to a very broad rainfall range.

A daily rainfall series is then obtained by aggregating the hourly series at a daily time-step. The simulation can be made long enough so that the distribution of rainfall can be approximated by the empirical distribution. From this empirical distribution, an estimate of the distribution of daily maximum rainfall is deduced. Extreme quantiles are computed from this estimated distribution. For example, a 100-year return level (which corresponds to the quantile of level 0.99) is determined by generating rainfall over a simulation period of 100 000 years.

3.2. Regional rainfall generator

The stochastic rainfall generator of SHYPRE can be extended to ungauged sites. For this, the parameters are first constrained and thus the number of free parameters is reduced. Second, the free parameters are interpolated spatially to allow the estimation at ungauged sites.

3.2.1. Constrained parametrization

The set of parameters is first constrained in the following way. The parameters which display little variability across various climates and thus across space or the parameters which do not influence much the rainfall simulations are assumed to take a single value for the whole region. After analysis on 217 pluviographic rain gauges in France, 15 parameters are fixed to the median value of the estimates at each rain gauge.

Daily rainfall series are much more widely available than hourly rainfall series. For this reason, we describe the five remaining parameters which belong to four variables as functions of daily rainfall. The first variable, the number of events per season, is already defined in terms of daily rainfall. We assumed that it follows a Poisson distribution and the parameter of its distribution, the expected value, is estimated directly from daily data. The parameters of the number of rainy periods within an event and the number of showers within a rainy period are parametrized in terms of a daily variable related to duration : the expected duration of an event in days. The parameters of the last variable, the cumulated rainfall in a shower, are parametrized in terms of a daily variable related to intensity : the expected maximum daily rainfall for the event in mm. The parametrizations are implemented with generalized linear models.

3.2.2. Spatial interpolation of daily rainfall variables

This constrained parametrization links hourly rainfall to daily rainfall features, that is the hourly rainfall generator can be parametrized from daily data. In order for the rainfall generator to work at ungauged sites, we need to interpolate spatially the daily rainfall variables. From these interpolated values and the generalized linear functions defined above, we can compute the parameters of the distributions of hourly variables while the other parameters are assumed to be identical across the region. In the next section, we explain how the spatial interpolation of the regional parameters for each method is performed and in particular, how the daily rainfall variables of the SHYPRE generator (expected number of events per season, expected maximum daily rainfall for the event and expected total duration of the event) are interpolated to ungauged sites.

4. Spatial interpolation

To build the interpolator functions, we combine multiple linear regression on geographic covariates together with a smoothing of the residuals.

4.1. Dimensionality reduction and variable selection

There are 22 potential covariates which describe the geographical space : latitude, longitude, altitude, 12 variables describing the landscape and seven distances to natural landmarks such as the sea, the Rhône river, the ocean, and so on [Sol and Desouches \(2005\)](#). For the RFA and the SIGEV methods, we reduce the dimensionality of the covariates (centered and scaled) by applying the Slice Inverse Regression (SIR) method [Li \(1991\)](#). SIR looks for a linear subspace of the covariates which has the best explanatory power. There is no assumption about the shape of the relationship between the covariates and the dependent variable. In practice, the SIR algorithm provides us with the orthonormal vectors which span the linear subspace (similarly to Principal Component Analysis). For the interpolation of the GEV parameters in SIGEV, we applied SIR with the 0.99 quantile as the dependent variable. Such a high-level quantile integrates the values of the three GEV parameters. In addition, this allows the definition a unique subspace for the SIGEV interpolation without resorting to a multivariate version of SIR. For the RFA method, we use the index value as the dependent variable.

In the SHYPRE method, from the pool of 22 potential geographic covariates, the group of three covariates which maximizes the explained variance, is selected. The selection is performed for each parameter being interpolated.

4.2. Index value interpolation

Let z_{i^*} be the dimension-reduced vector of geographic covariates at site i^* . The interpolator function takes the following shape for the index value parameter in Eq.(9) :

$$m(z_{i^*}) = \exp(z_{i^*}'\beta + \varepsilon_{i^*}), \quad (10)$$

where β is a vector of regression coefficients and ε_{i^*} is estimated by smoothing the regression residuals with kernel regression [Bishop \(1995\)](#) :

$$\widehat{\varepsilon}_{i^*} = \frac{\sum_{i=1}^{n_{\text{train}}} K_h(d_{i,i^*}) \varepsilon_i}{\sum_{i=1}^{n_{\text{train}}} K_h(d_{i,i^*})} \quad (11)$$

where d_{i,i^*} is the Euclidean distance between sites i and i^* in the x-y coordinate plane, n_{train} is the number of observations in the training set, $K_h(d) = \exp(-d^2/2h^2)$ is the Gaussian kernel and the bandwidth is fixed to $h = 10$ km. Residuals are thus smoothed at site i^* by considering mainly the stations within a 20 km circular radius with increasing weights to closest stations. The relationship in Eq.(10) is taken to be log-linear to ensure the positivity of the index value parameter.

4.3. Local GEV parameters interpolation

For SIGEV, we have a multivariate regression, see Eq.(3) :

$$\begin{pmatrix} \mu(z_{i^*}) \\ \sigma(z_{i^*}) \\ \xi(z_{i^*}) \end{pmatrix} = g(z_{i^*}' \alpha + v_{i^*}), \quad (12)$$

where α is a matrix of regression coefficients and v_{i^*} is a vector of length three which is estimated by smoothing independently the regression residuals with kernel regression, see Eq.(11). The same bandwidth parameter, $h = 10$ km, is applied each time. Similarly to the exponential function in Eq.(10), the function $g(\cdot)$ enforces positivity constraints for the location and scale parameters and is defined as $g(a_1, a_2, a_3) = (\exp a_1, \exp a_2, a_3)$.

4.4. Daily rainfall variables interpolation

The regional parameters of the SHYPRE method are the daily rainfall variables (see section 3.2.1) which are interpolated with multiple regression together with a smoothing of the residuals. The smoothing is performed according to the inverse squared distance.

4.5. Intrinsic parameters

The two regional methods based on Extreme Value Theory have intrinsic parameters which must be estimated from the stations in the training set in order to obtain the distribution of maximum rainfall at each site i^* of the n_{valid} sites in the validation set. For SIGEV, the intrinsic parameters include only the regression coefficients $\phi^{\text{SIGEV}} = \alpha$ and, for the RFA method, these include also the regional GEV parameters at the sites from the validation set $\phi^{\text{RFA}} = \{\beta, (\mu_{i^*}^R \sigma_{i^*}^R, \xi_{i^*}^R)_{i^*=1}^{n_{\text{valid}}}\}$.

In the SHYPRE regional rainfall generator, the constrained parameters are calibrated once and for all on the 217 pluviographic rain gauges. The intrinsic parameters of the SHYPRE generator which must be estimated in order to obtain an empirical estimate of the daily rainfall distribution at a new site i^* are the regression coefficients which we write ϕ^{SHYPRE} .

5. Rainfall Data and Evaluation Framework

We have daily observations from 1046 rain gauge stations gathered from the French Weather Service "Météo France" and the Electricity Society "Électricité de France". The stations are located in the South of France, from the Atlantic ocean near Spain, to the Mediterranean coast, all the way up to the Alps, near the Italian border. For the regional methods based on Extreme Value Theory (RFA and SIGEV), we are interested in daily annual maxima over the hydrological year, which starts June 1st and ends May 31st. In order to have reliable maxima, years containing more than 10% missing observations were screened out. Thus, there are between 2 to 58 years of observed daily annual maxima. For the SHYPRE method, months with more than 10 days without observations are considered as missing as a whole. Globally, SHYPRE is thus able to use more data for the estimation of its parameters.

5.1. Performance evaluation

The 1046 stations are split into a training set and a validation set. Figure 3a depicts the split into a 50% training set (in blue dots) and 50% validation set (in red squares). Four additional training/validation splits are considered : 33% training - 66% validation, 25% training - 75% validation plus two other splits, A and B, which aim at challenging the regional methods. In the training set A, there are 609 stations with short observation period, from 2 to 34 years. In training set B, the challenge is made even more difficult by considering only 411 stations among the 609 from training set A. The validation set in both cases contains 411 stations with a least 46 years of observations. In all five cases, the splits are made so as to ensure a minimum spatial coverage.

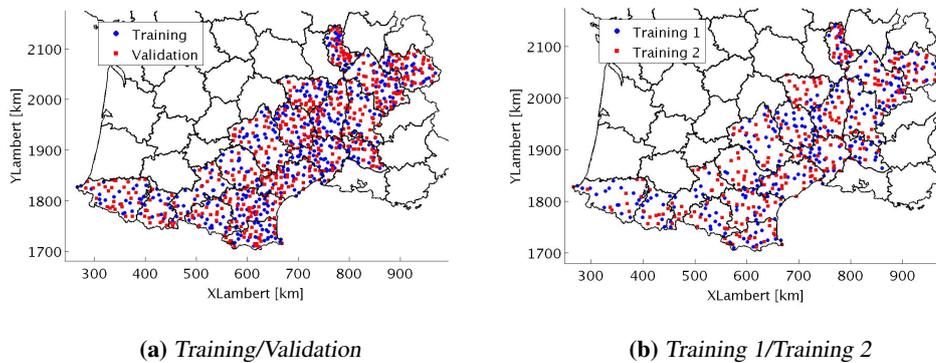


Figure 3: 1046 stations with 2 to 58 years of observations.

The training/validation splits are defined in order to evaluate the performance of the regional methods. The SIR projection vectors and the parameters of the spatial interpolator functions (the regression coefficients (β in Eq.(10), α in Eq.(12)) and those for the interpolation of the SHYPRE daily rainfall variables) are computed on the training set. The interpolator functions are then applied to the covariates of the validation set to estimate the index value, the local GEV parameters and the SHYPRE daily rainfall variables at each site of the validation set. The homogeneous neighborhood for RFA is defined for each station in the validation set exclusively from the stations in the training set. In other words, the stations in the validation set are considered ungauged and thus their observations are not used for training, only for validation. Accordingly, the regional GEV parameters in Eq.(7) are estimated with the observations from the stations in the training set.

Two performance criteria are designed to evaluate the goodness-of-fit of each method on the observations from the stations in the validation set with particular care for the extreme part of the distribution. These criteria do not rely on standard statistical tests so as to circumvent the pitfall of the spatial dependence for nearby stations and they do not involve the empirical frequency of large observations to avoid the variability and the extrapolation issue in small samples. See Renard et al. (2013) for details on these criteria.

5.1.1. *Quantile violations*

The first criterion is based on the number of quantile violations : it verifies that the number of observations above a quantile estimated by a given method is consistent with the quantile level (if we estimate a 90%-quantile, when the model is right, we expect to have approximately 10% of the observations above this quantile). More precisely, for a given regional method and a given training/validation split, let $\hat{\phi}$ be the so-called intrinsic parameter vector (see section 4.5) estimated on the training set. Let $q_{i^*}^{\hat{\phi}}(T)$ be the return level estimate with return period T years (see Eq.(2)) at site i^* of the validation set computed with the parameters $\hat{\phi}$. The number of quantile violations $n_{i^*}^{\hat{\phi}}(T)$ corresponds to the number of observations at station i^* greater than $q_{i^*}^{\hat{\phi}}(T)$. If the quantile is accurately estimated, we have that :

$$\mathcal{Q}_T : n_{i^*}^{\hat{\phi}}(T) \sim \mathcal{B}(n_{i^*}, 1/T) \tag{13}$$

where n_{i^*} is the number of observations at site i^* and \mathcal{B} indicates the Binomial distribution with number of trials n_{i^*} and success probability $1/T$.

5.1.2. *Maximum of maxima*

The second criterion is based on the maximum observation $X_{\max i^*} = \max_{k=1, \dots, n_{i^*}} X_k$, for a given station i^* . Let $\hat{F}_{i^*}^{\hat{\phi}}$ be the distribution function of annual maximum rainfall at site i^* estimated by a given regional method with intrinsic parameters $\hat{\phi}$. Then we have that :

$$\mathcal{M} : \hat{F}_{i^*}^{\hat{\phi}}(X_{\max i^*}) \sim \mathcal{K}(n_{i^*}, 1) \tag{14}$$

where $\mathcal{K}(n_{i^*}, 1)$ is the Kumaraswamy distribution **Kumaraswamy (1980)**. In other words, if the model is correct, we should have that $P(\hat{F}_{i^*}^{\hat{\phi}}(X_{\max i^*}) \leq x) = x^{n_{i^*}}$.

5.1.3. *Graphical and numerical evaluation*

The validity of hypotheses \mathcal{Q}_T in Eq.(13) for return periods of $T = 5$ and $T = 10$ years and \mathcal{M} in Eq.(14) is checked for each regional method and each training/validation splits. We first looked at probability-probability plots (p-p plots for short) to evaluate globally the validity of \mathcal{Q}_T and \mathcal{M} . Let $B_{n_{i^*}, T}$ and $K_{n_{i^*}}$ be the distribution functions of the binomial and Kumaraswamy distributions respectively. Under the hypothesis that the model is correct, the cumulative frequencies $K_{n_{i^*}}(\hat{F}_{i^*}^{\hat{\phi}}(X_{\max i^*}))$ should be uniformly distributed. For the binomial cumulative frequencies, $B_{n_{i^*}, T}(n_{i^*}^{\hat{\phi}}(T))$, the distribution, under the correct model hypothesis, is discrete and asymmetric for large values of T . This distribution is smoothed by sampling uniform variates in the interval $[B_{n_{i^*}, T}(n_{i^*}^{\hat{\phi}}(T) - 1), B_{n_{i^*}, T}(n_{i^*}^{\hat{\phi}}(T))]$ and then should, if the model is appropriate, follow a uniform distribution. The p-p plot relates the cumulative frequencies from the model to the empirical frequencies which assumes a uniform distribution. If the plot is close to the diagonal line, this supports the validity of the hypothesis of model correctness.

Secondly, we computed a criterion which measures how close the p-p plot is from the diagonal line :

$$C^{\hat{\phi}}(\mathcal{Q}_T) = 1 - 2 \frac{1}{n_{\text{valid}}} \sum_{i^*=1}^{n_{\text{valid}}} |B_{n_{i^*}, T}(n_{i^*}^{\hat{\phi}}(T)) - \hat{H}_{i^*}| \quad (15)$$

$$C^{\hat{\phi}}(\mathcal{M}) = 1 - 2 \frac{1}{n_{\text{valid}}} \sum_{i^*=1}^{n_{\text{valid}}} |K_{n_{i^*}}(\hat{F}_{i^*}^{\hat{\phi}}(X_{\max i^*})) - \hat{H}_{i^*}| \quad (16)$$

where n_{valid} is the total number of sites in the validation set and \hat{H}_{i^*} is the appropriate empirical frequency. When $C^{\hat{\phi}}(\cdot) = 1$ this means perfect fit whereas $C^{\hat{\phi}}(\cdot) = 0$ means bad fit.

5.2. Sensitivity to training data

In addition to the training/validation splits, pairs of training sets are defined in order to assess the sensitivity of each regional method to the training set. Fig. 3b illustrates one such pair with medium spatial coverage, that is there are 309 stations in each training set which are chosen so as to cover the whole region. Two other pairs are defined with low spatial coverage (154 stations) and full spatial coverage (522 stations). The sensitivity is measured in terms of *span* of high-level quantile estimates computed for fictitious validation sites which are taken to lie on a 1 km grid covering the region. For a given method, let $\hat{\phi}_1$ and $\hat{\phi}_2$ be the intrinsic parameters estimated on the first and the second training set respectively. For a given validation site i^* , let $q_{i^*}^T(\hat{\phi}_1)$ and $q_{i^*}^T(\hat{\phi}_2)$ be the return levels with period T computed with the parameters estimated on the first and the second training set respectively. Then the *span* is defined as :

$$\mathcal{S}_{i^*}^T(\hat{\phi}_1, \hat{\phi}_2) = \frac{|q_{i^*}^T(\hat{\phi}_1) - q_{i^*}^T(\hat{\phi}_2)|}{[q_{i^*}^T(\hat{\phi}_1) + q_{i^*}^T(\hat{\phi}_2)]}. \quad (17)$$

When $\mathcal{S}_{i^*}^T(\hat{\phi}_1, \hat{\phi}_2) = 0$, the regional method is not sensitive to the training set as it yields always the same return level estimates. As the *span* increases, it indicates a greater sensitivity to the training set. The *span* is bounded above by 1; this can be seen as $T \uparrow \infty$ with $q_{i^*}^T(\hat{\phi}_1)$ is bounded above, but $q_{i^*}^T(\hat{\phi}_2) \uparrow \infty$ when $T \uparrow \infty$. To summarize the *span* over all the validation grid, we look at the average *span* :

$$\overline{\mathcal{S}^T}(\hat{\phi}_1, \hat{\phi}_2) = \frac{1}{n_{\text{valid}}} \sum_{i^*=1}^{n_{\text{valid}}} \mathcal{S}_{i^*}^T(\hat{\phi}_1, \hat{\phi}_2) \quad (18)$$

where $n_{\text{valid}} = 102734$ is the number of grid boxes on the the 1 km grid validation set.

6. Results

For RFA and SIGEV, over all training sets, a SIR-subspace of dimension six to eight (depending on the training set) is required to explain 95% of the variance of the dependent variable in the spatial interpolator functions, see section 4. We first compare the regional methods for each performance criterion on all training/validation splits in section 6.1, then the comparisons are

made in terms of training/validation splits for all performance criteria in section 6.2 and finally, the sensitivity to the training set is evaluated in section 6.3.

6.1. Comparisons per criterion

In Fig. 4, we can assess how the training/validation splits affect each regional method (RFA in the left column, SIGEV in the middle column and SHYPRE in the right column) in terms of the quantile violation criterion \mathcal{Q}_T for 5, and 10 years (top and middle rows) and in terms of maximum of maxima criterion \mathcal{M} (bottom row).

The RFA method is generally less affected by the challenging training/validation splits A and B than SIGEV. Despite short observations period of training sets A and B, RFA increases the sample size by including observations in the homogeneous neighborhood, whereas SIGEV is limited to the observations in each station. In addition, RFA performs better at capturing higher quantiles ($T = 10$) than lower quantiles ($T = 5$). However, SIGEV performs surprisingly well in terms of \mathcal{M} despite relying only on local information. Yet, its performance deteriorates noticeably for the short observation period and smaller number of stations of split B. On the other hand, the performance of the SHYPRE method is unaffected by the training set since all the p-p curves are grouped together, whatever the return level. Similarly to RFA, the performance of SHYPRE decreases for lower quantiles.

6.2. Training/Validation split comparisons

The comparisons for all three criteria \mathcal{Q}_5 , \mathcal{Q}_{10} and \mathcal{M} for each split are depicted in Fig. 5. The spider plots rely on the measure of distance $C^{\hat{\phi}}$ between the p-p plots and the diagonal line defined in Eqs.(15-16).

The performance of SHYPRE and RFA for the \mathcal{Q}_{10} criterion is higher than for the SIGEV method. The same is true for \mathcal{Q}_5 except for the first C/V split (C25 / V75) for which SHYPRE is worst than SIGEV. For the \mathcal{M} criterion, the performances of the three methods are variable except for the C/V split B. For the latter, which is the most challenging C/V split, we clearly see that the SHYPRE and RFA methods outperform the SIGEV method for \mathcal{Q}_5 , \mathcal{Q}_{10} and \mathcal{M} .

6.3. Sensitivity to training data

The sensitivity is evaluated thanks to the average *span*, see Eq.(18), represented in Fig. 6, for return periods $T = 10$, $T = 100$ and $T = 1000$ for the three pairs of training sets (low, medium and full spatial coverage). The sensitivity is higher for sparser training set (lower spatial coverage) and for higher return periods, as expected. SIGEV is the method most sensitive to the spatial coverage since its average span, as a measure of sensitivity, decreases steadily with increasing spatial coverage. SHYPRE and RFA have a sharp decrease in average span from the low to the medium spatial coverage. However, there is only a tiny decrease in sensitivity when we switch from medium to full spatial coverage. For the return period of $T = 10$, RFA and SIGEV are very similar whereas SHYPRE has a higher average span. For the return period of $T = 100$, the ordering in terms of increasing sensitivity is : RFA, then SIGEV and last SHYPRE. Finally for $T = 1000$, SIGEV has a larger sensitivity whereas RFA and SHYPRE are similar.

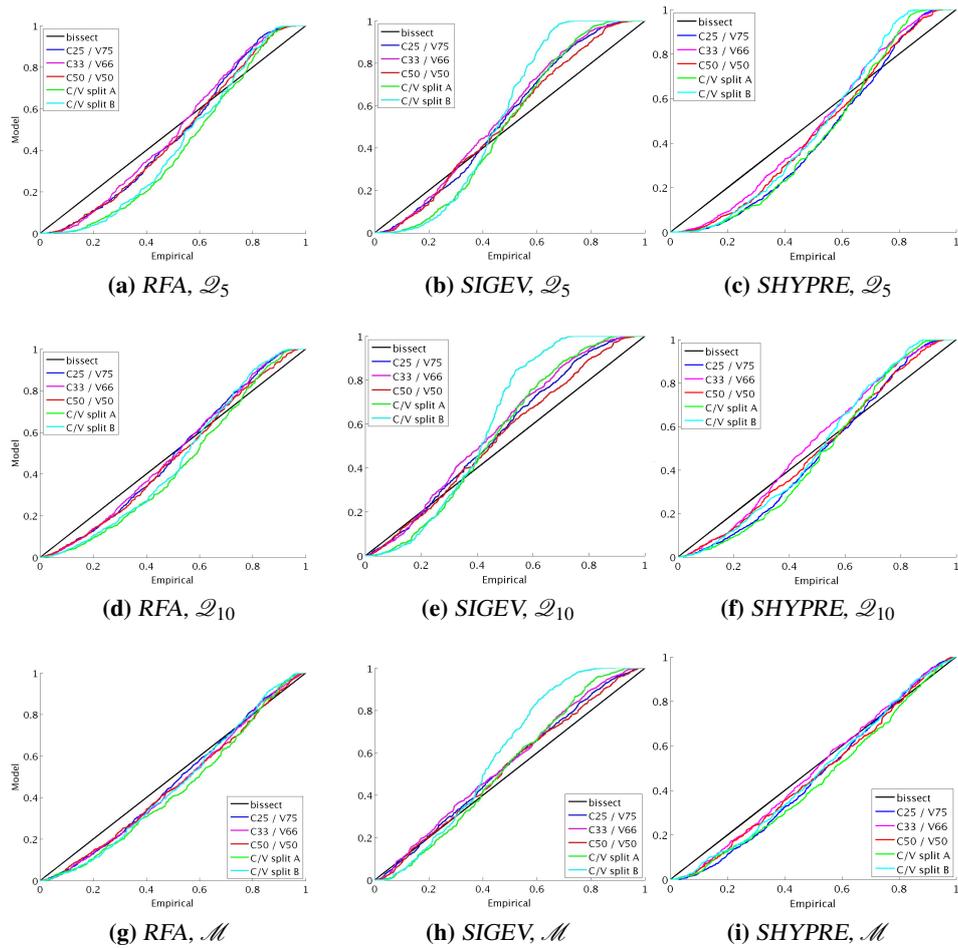


Figure 4: Quantile violations p - p plots for return levels 5 and 10 years (top and middle rows) and maximum of maxima p - p plots (bottom row) on all five train/validation splits for the regional frequency analysis method (left), the spatial interpolation method (middle) and the SHYPRE method (right).

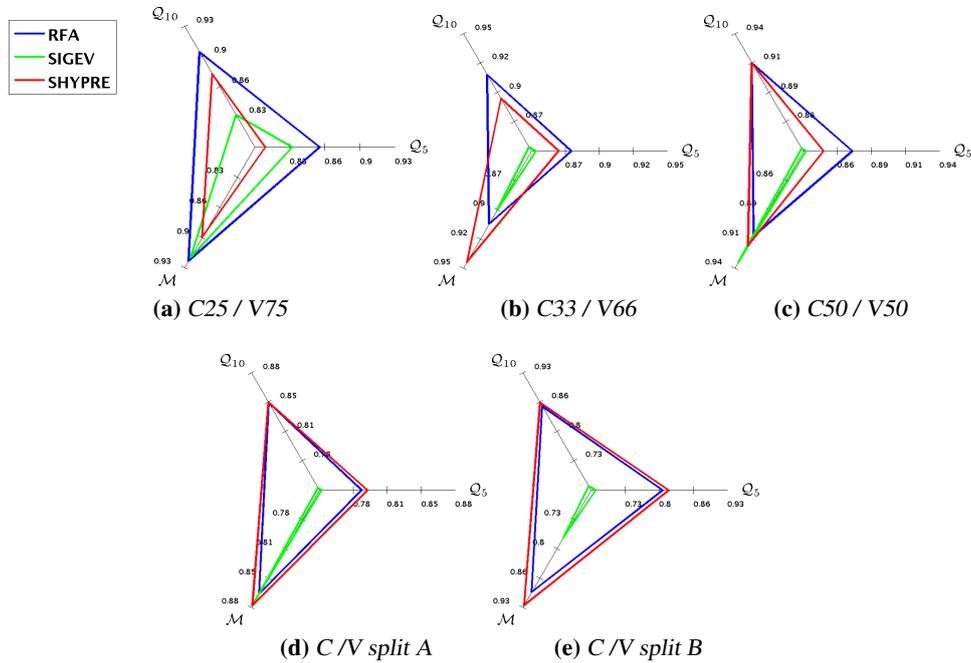


Figure 5: Comparisons of RFA, SIGEV and SHYPRE on each training/validation splits for the quantile violations ($C^{\hat{\phi}}(\mathcal{Q}_5)$ and $C^{\hat{\phi}}(\mathcal{Q}_{10})$ for return periods 5 and 10 years respectively) and the maximum of maxima ($C^{\hat{\phi}}(\mathcal{M})$).

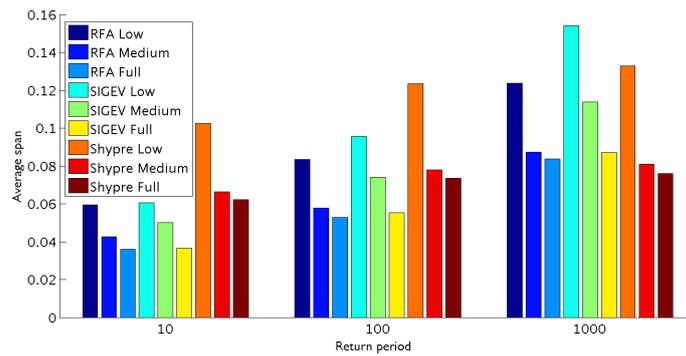


Figure 6: Average span for return periods $T = 10$, $T = 100$ and $T = 1000$ years for pairs of training sets with low, medium and full spatial coverage evaluated on a 1 km grid covering the region.

7. Discussion and Conclusion

We compared two families of regional methods which address the question of extreme rainfall estimation in completely different ways. On one hand, there are two regional methods (SIGEV and RFA) which rely on the asymptotic behaviour of extreme events to approximate their finite behaviour. The Generalized Extreme-Value Distribution parameters are estimated, either locally or regionally, and extreme return levels are estimated from this distribution. SIGEV interpolates spatially the three local GEV parameter estimates and is thus sensitive to the size of the local sample. RFA makes the assumption of a homogeneous neighborhood around the site of interest and the observations from all the sites in the neighborhood contribute to the estimation of the regional GEV parameters. This increased sample size, compared to the local sample size, decreases the variance of the shape parameter estimate. In addition, only one parameter, the index value, has to be spatially interpolated. However, if the homogeneity assumption is not fulfilled, the regional approach might introduced bias in the GEV parameter estimates.

On the other hand, the SHYPRE method is based on the characterization of various features of rainfall events to build a stochastic hourly rainfall generator from which return levels can be estimated empirically. SHYPRE has evolved considerably over the years in order to improve its description of rainfall events while keeping a parsimonious parametrization.

The evaluation framework applied here draws from [Renard et al. \(2013\)](#). It consists in two types of performance measures : the fitness of the quantile estimates through the number of quantile violations and the fitness of the probability attributed to the maximum of the annual maxima. The evaluation framework also includes a measure of sensitivity to the training set which is the average span between extreme quantile estimates computed from a pair of training sets. The goal of the evaluation framework is to compare the regional methods without relying on parametric hypothesis testing, to avoid the complexity involved with spatial dependencies and to focus on extreme rainfall.

We draw the following main conclusions from the comparisons conducted in this work. The performance results presented above do not highlight a clear preferred regional method. SIGEV, which can be thought of as the benchmark method, is to be applied with caution for sparsely sampled data sets. Indeed, SIGEV is sensitive to the size of the rainfall samples which affect the variance of the local GEV parameter estimates. This is the case for the training/validation split B where the sample size is small and SIGEV's performance deteriorates. The sensitivity of the SIGEV method is also noticeable from its high average span for the 1000-year return level and lower spatial coverage pair of training sets (see Fig. 6). A possibility to increase the robustness of SIGEV along the lines of the RFA approach would be to keep the tail index constant across the region. However, this seems like a fairly restrictive and unrealistic assumption given the variability of rainfall extremes in the region. A middle ground solution would be to find sub-regions with constant tail index parameter but this is exactly what RFA does.

The performance of the SHYPRE method is very stable across data sets. It might be thanks to the relationships between hourly and daily variables which are derived a priori on separate data sets. Also, the SHYPRE method has overall higher average span but this is due to the spatial interpolation of its parameters which could be improved. Finally, we found that, overall, RFA and SHYPRE, two very different approaches, lead to similar performance. One point in favor of RFA is its ease of implementation. In favor of SHYPRE, simulations at sub-daily time step, e.g. hourly

time-step, are also provided. The key issue and challenge regarding these three regional methods is a proper spatial interpolation of their parameters.

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